Determining the charge of an electron using a computer simulation

Introduction

Elementary charge (also known as fundamental charge) refers to the electrical charge of a single electron, and the "smallest amount of charge that exists in ordinary matter"[1].

According to the International Union of Pure and Applied Physics[2], the elementary charge (*e*) is approximately 1.602×10^{-19}C. When I first encountered this value in class, I wondered how this seemingly arbitrary number was acquired and why we universally accept it as the charge of the electron, especially since the electron is so microscopic and mysterious, even today, more than a century after its discovery. This curiosity prompted me to investigate how the existence and value of the elementary charge was determined.

Historical Background

The existence of electron and its charge-to-mass ratio was discovered by J.J. Thomson in 1897 through his cathode ray experiment. However, this did not signify that the charge nor mass of the electron was fundamental or unique. Many attempted to find the charge of the electron by examining clouds of water droplets[3], but their results were not convincing because they relied on averages.

In the 1900s, Robert Millikan and his students realized that increasing the electric field could isolate a few droplets of water but this was unsuccessful as the water evaporated too quickly. Thus, they modified the experiment to using droplets of oil. Millikan's apparatus is shown in Figure 1. Over several years, Millikan's oil-drop experiment was performed and improved upon, until finally, in 1913, a value of the elementary charge was discovered with an "error bar of just 0.2 percent"[4]. This is a highly precise number and demonstrates how amazing this experiment was.

Figure 1 Millikan's oil-drop apparatus

Research Question

The primary aim of this investigation is to recreate the Millikan oil-drop experiment using a computer simulation and determine the value for the charge of an electron through two different methods, Millikan's original method by measuring the voltage at which the oil drop is stationary, and an alternative method by measuring the terminal velocity of the oil drop at a constant voltage.

I chose to use a computer simulation because this experiment involves specialized and expensive equipment that were not available to me. Furthermore, using a computer simulation provides me with more accuracy and precision for this delicate experiment.

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1 Hamper, Chris, and J. K. Ord. *Physics : standard level : developed specifically for the IB diploma*. Harlow, Essex: Pearson Education, 2007. Print.

2 Cohen, E. Richard, and Pierre Giacomo. *Symbols, units, nomenclature and fundamental constants in physics*. North-Holland, 1987.

3 Schirber, Michael. "Millikan Measures the Electron's Charge." APS Physics. 2012. Web. 22 Mar. 2016. <http://physics.aps.org/articles/v5/9>.

4 Ibid.

Millikan Oil-Drop Experiment

Design and Methodology

The set-up of Millikan's experiment is shown in Figure 2:

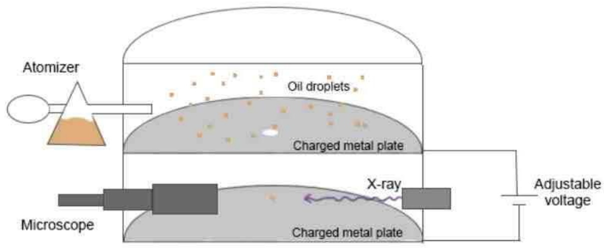

Figure 2 Design of Millikan's experiment

Millikan's apparatus consisted of two chambers, separated by a metal plate with a small pinhole. The two charged metal plates were set up horizontally with an electric field between them that could be adjusted. A thin mist of oil was sprayed into the top chamber using the atomizer and droplets fell through the hole. Without the electric field, the droplet would quickly reach a constant velocity. Millikan let them fall with no electric field in order to measure their terminal velocity and then, calculated the mass of each oil droplet. The forces acting on the oil droplets are the weight of droplets, upthrust and air resistance[5]. Since the velocity is equal, this means that the net force acting on the droplet is 0.

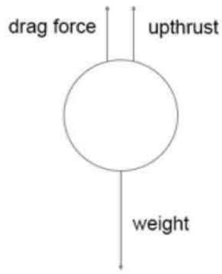

Figure 3 Forces acting on a free-falling oil droplet

After he found the mass of a single droplet, he ionized the droplets by using the X-ray radiation to allow electrons from the air to attach to the oil. The voltage of the electric field could be adjusted and the speed of the droplet's fall could be increased or decreased, and it could eventually start to move upward if the upward electric force was high enough[6]. With the introduction of an electric field, the electric force also acts on the droplet in the upward direction. Since the droplet starts to move upwards, the drag force goes in the downward direction. Millikan carefully found the voltage of the electric field at which the oil droplet could float stationary in the bottom chamber. At this voltage, the forces acting on the droplet (weight and electric force) cancel each other out. The amount of voltage needed to suspend the oil droplet is used to find the electric charge on the droplet. After this experiment was repeated hundreds of times, Millikan noticed that the charges of the individual oil drops were always a multiple of the lowest charge, approximately 1.602×10^{-19}C. This was identified to be the charge of an electron.

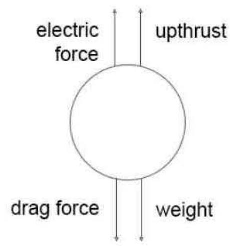

Figure 4 Forces acting on an oil droplet in an electric field

Finding the charge of the electron was a ground-breaking discovery because with this information, Millikan was later able to find the mass of an electron, using the charge-to-mass ratio that J.J. Thomson determined in the late 19th century.

Selecting a Simulation

In my search for the simulation best suited for this investigation, I was searching for a program that closely resembled the actual hands-on experiment. I needed a detailed program that allowed me to measure the velocity and voltage accurately. The first simulation I encountered was one from The King's Centre for Visualization in Science. I immediately noticed that this simulation was very simplistic. There were no graduations on the simulation, so I could not determine the distance that the oil droplet travelled in a given time, nor find the velocity of the free-falling droplet. So, I continued my search. I found a total of 6 simulations before I settled on the one that met my

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[5] The Editors of Encyclopædia Britannica. "Millikan Oil-drop Experiment." Encyclopedia Britannica Online. Encyclopedia Britannica. Web. 23 Mar. 2016.

[6] Ibid.

personal criteria. This was the one from the Université de Saint-Boniface. This program has coarse tuning and fine tuning of the voltage, which can provide more precise results. In addition, there is a built-in ruler and stopwatch that can help me take measurements of the distance and duration. This was an unique program because it allowed me to set up the reticle, level, focus, and magnification of the apparatus, providing a more interactive and hands-on experience, though it is a simulation. Furthermore, the program provided me with information about the density of the air and oil, and the radius of the oil droplets, which aided me in my calculations. Figure 5 is a screenshot of the simulation.

Figure 5 Simulation from the Université de Saint-Boniface

Method 1

I first used the simulation to recreate the original method that Millikan used to find the electrical charge.

Deriving Equations

Figure 6 shows the forces acting on the droplet. In Millikan's original experiment, he found the voltage at which the droplet hovers stationary in the chamber, so according to Newton's second law, the forces would be balanced:

Drag force (F_d)+ Weight (W) = Upthrust (F_u) + Electric force (E)

The weight of the droplet is $W = mg$ and mass can be found by the density equation:

$$density = \frac{mass}{volume}$$

$$\Rightarrow m = volume \times density$$

$$\Rightarrow m = \frac{4}{3}\pi r^3 \rho_{oil}$$

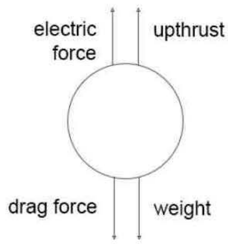


Figure 6 Forces acting on an oil droplet in an electric field

Hence, the weight of the droplet is $W = \frac{4}{3}\pi r^3 \rho_{oil} g$; where ρ_{oil} is density of oil, and r is radius of droplet.

Upthrust is the upward force acting on the droplet by the air due to the displacement of air by the oil droplet, and is $F_u = \frac{4}{3}\pi r^3 \rho_{air} g$; where ρ_{air} is density of air.

Electric force is $F_e = qE$ where q is charge, and E is electric field. The electric field is equivalent to $\frac{V}{d}$; where V is voltage, and d is distance between the two charged metal plates in the apparatus.

Finally, the drag force is the force acting in the opposite direction of an object's motion through a viscous fluid. This is given by Stokes' Law to be $F_d = 6\pi\eta r v_e$; where η is viscosity, and v_e is terminal velocity of droplet. Since the droplet is stationary, its velocity is 0 and so, the drag force acting on the droplet is 0.

So the equation can be rewritten as:

$$\frac{4}{3}\pi r^3 \rho_{oil} g = \frac{4}{3}\pi r^3 \rho_{air} g + qE$$

Solving for q:

$$\frac{4}{3}\pi r^3 \rho_{oil} g = \frac{4}{3}\pi r^3 \rho_{air} g + qE$$
$$\Rightarrow \frac{4}{3}\pi r^3 \rho_{oil} g - \frac{4}{3}\pi r^3 \rho_{air} g = qE$$
$$\Rightarrow \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g = qE$$
$$\Rightarrow q = \frac{4}{3E}\pi r^3 (\rho_{oil} - \rho_{air}) g$$

Substituting E with $\frac{V}{d}$:

$$q = \frac{4}{3E}\pi r^3 (\rho_{oil} - \rho_{air}) g$$
$$\Rightarrow q = \frac{4d}{3V}\pi r^3 (\rho_{oil} - \rho_{air}) g$$

From the formula above, given the density of oil and air, radius of the droplet, and distance between the plates, I can solve for the value of q by recording values for V. The program I used provided me with the parameters of the simulation:

- Density of oil: ρ_{oil} = 1050 kg m^{-3}
- Density of air: ρ_{air} ≈ 1.2 kg m^{-3}
- Radius of the droplet: r= 4.57×10^{-7} m
- Distance between the plates: d = 4 mm

Raw Data

I used the sprayer to spray a bead of oil into the main screen and then used the course tuning of the voltage of the electric field until the bead slowed down significantly. I then fine-tuned the voltage until the bead was completely stationary, even at 32x magnification, and recorded the voltage in an Excel spreadsheet. I repeated this for 15 trials.

Trial	**Voltage (± 0.01V)**
1	12.86
2	51.44
3	20.60
4	25.72
5	14.70
6	10.29
7	102.90
8	34.24
9	102.81
10	102.86
11	17.14
12	20.57
13	24.26
14	10.28
15	51.40

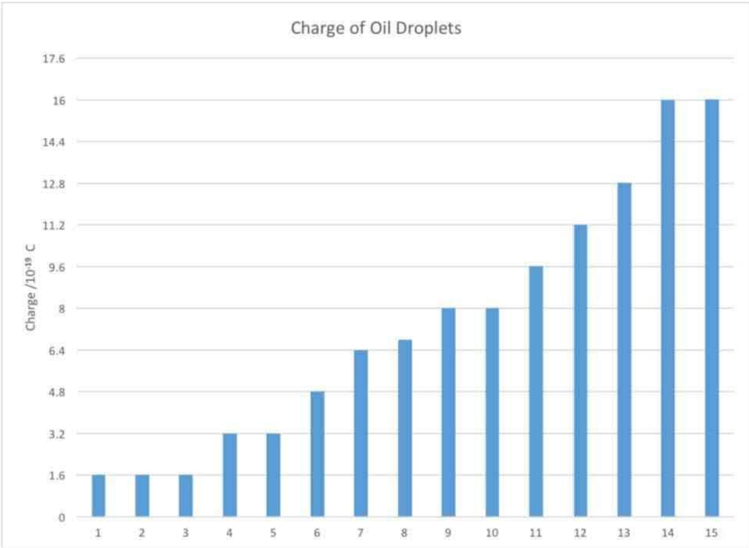
Processed Data
With a spreadsheet in Excel, I substituted the voltage and other given values into the final equation I derived to calculate the charges of each trial. The uncertainty in the voltage is omitted in the calculation of charge because the relative uncertainty is so low that it is insignificant.

Trial	Voltage (± 0.01V)	Charge (10^{-19} C)
1	12.86	12.81
2	51.44	3.202
3	20.60	7.996
4	25.72	6.404
5	14.70	11.21
6	10.29	16.01
7	102.90	1.6008
8	34.24	4.811
9	102.81	1.60212
10	102.86	1.6014
11	17.14	9.610
12	20.57	8.008
13	24.26	6.790
14	10.28	16.02
15	51.40	3.205

To better demonstrate the clusters of droplets with the same charge, I sorted the chart by the order of least to greatest charge.

Voltage (± 0.01V)	Charge (10^{-19} C)
102.90	1.6008
102.86	1.6014
102.81	1.6022
51.44	3.202
51.40	3.205
34.24	4.811
25.72	6.404
24.26	6.790
20.60	7.996
20.57	8.008
17.14	9.610
14.70	11.21
12.86	12.81
10.29	16.01
10.28	16.02

By graphing the charges in a bar graph, the common factor of approximately 1.6×10^{-19} is obvious, as shown by the distinct stair pattern.

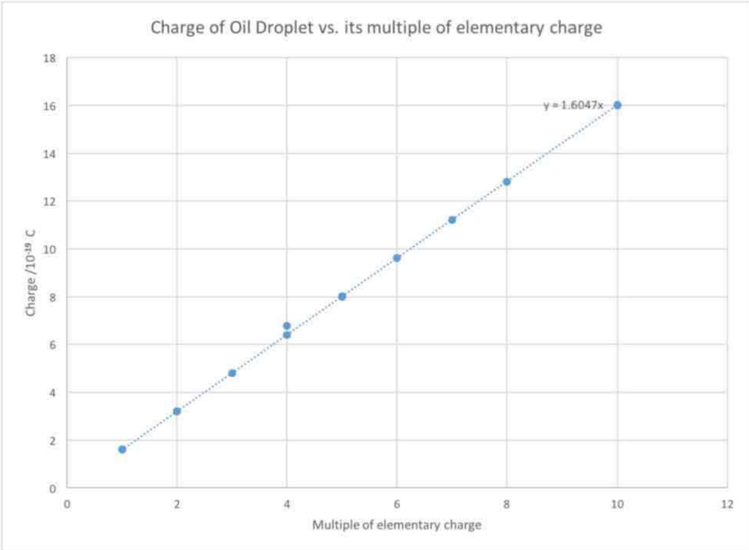


Each result appears to be a multiple of the lowest value of charge, approximately 1.6×10^{-19}C. To find the value that of the elementary charge, I grouped the charges by their apparent multiples of the elementary charge. This results in the equation:

$q = multiple\ of\ e\ \times\ elementary\ charge$

Voltage (± 0.01V)	**Charge (10^{-19} C)**	**Multiple of elementary charge**
102.90	1.6008	1
102.86	1.6014	1
102.81	1.6022	1
51.44	3.202	2
51.40	3.205	2
34.24	4.811	3
25.72	6.404	4
24.26	6.790	4
20.60	7.996	5
20.57	8.008	5
17.14	9.610	6
14.70	11.21	7
12.86	12.81	8
10.29	16.01	10
10.28	16.02	10

I graphed the charge and the multiple of q in a scatter graph to find the gradient of the line of best fit, which represents the value of elementary charge.



According to my results, the elementary charge of an electron is approximately 1.6047×10^{-19}C.

Method 2

After finding such a fascinating and versatile simulation, I was excited to further explore other methods of finding elementary charge. I noticed that if the voltage supplied was sufficient, oil droplets will start to rise and reach a terminal velocity. By comparing the terminal velocity of a bead of oil rising with the presence of the electric field and free-falling in its absence, the charge of the droplet can be determined.

Deriving Equations

Once again, I use the same equations and values as in method 1.

- Weight of droplet: $W = \frac{4}{3}\pi r^3 \rho_{oil} g$
- Upthrust: $F_u = \frac{4}{3}\pi r^3 \rho_{air} g$
- Electric force: $F_e = qE$
- Electric field: $E = \frac{V}{d}$
- Drag force: $F_d = 6\pi\eta r v_e$
- Density of oil: $\rho_{oil} = 1050$ kg m^{-3}
- Density of air: $\rho_{air} \approx 1.2$ kg m^{-3}
- Radius of the droplet: $r = 4.57 \times 10^{-7}$ m
- Distance between the plates: $d = 4$ mm

The net force acting on the droplet is
Weight (W) + Drag force (F_d)= Upthrust (F_u) + Electric force (E)

$$\frac{4}{3}\pi r^3\rho_{oil}g + 6\pi\eta r v_e = \frac{4}{3}\pi r^3\rho_{air}g + qE$$

I had no information about the viscosity of the air within the chamber for this simulation, so I derive an equation for viscosity using the following equation[7]:
$v_g = \frac{2r^2(\rho_{oil}-\rho_{air})g}{\eta} \Rightarrow \eta = \frac{2r^2(\rho_{oil}-\rho_{air})g}{9v_g}$; where v_g is the terminal velocity under free-fall.

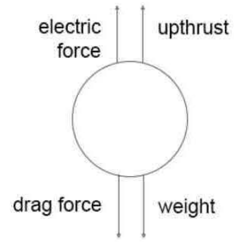


Figure 7 Forces acting on an oil droplet in an electric field

Substituting η into the equation for the forces acting on the oil droplet and solving for q:

$$\frac{4}{3}\pi r^3\rho_{oil}g + 6\pi(\frac{2r^2(\rho_{oil}-\rho_{air})g}{9v_g})rv_e = \frac{4}{3}\pi r^3\rho_{air}g + qE$$

$$\Rightarrow qE = \frac{4}{3}\pi r^3\rho_{oil}g + 6\pi\left(\frac{2r^2(\rho_{oil}-\rho_{air})g}{9v_g}\right)rv_e - \frac{4}{3}\pi r^3\rho_{air}g$$

$$\Rightarrow qE = \frac{4}{3}\pi r^3(\rho_{oil}-\rho_{air})g + 6\pi\left(\frac{2r^2(\rho_{oil}-\rho_{air})g}{9v_g}\right)rv_e$$

$$\Rightarrow qE = \frac{4}{3}\pi r^3(\rho_{oil}-\rho_{air})g + \frac{12\pi r^3(\rho_{oil}-\rho_{air})gv_e}{9v_g}$$

$$\Rightarrow qE = \frac{4}{3}\pi r^3(\rho_{oil}-\rho_{air})g + \frac{4\pi r^3(\rho_{oil}-\rho_{air})gv_e}{3v_g}$$

$$\Rightarrow qE = \frac{4}{3}\pi r^3(\rho_{oil}-\rho_{air})g + (1+\frac{v_e}{v_g})$$

$$\Rightarrow q = \frac{4}{3E}\pi r^3(\rho_{oil}-\rho_{air})g + (1+\frac{v_e}{v_g})$$

Substituting E with $\frac{V}{d}$:

$$q = \frac{4}{3E}\pi r^3(\rho_{oil}-\rho_{air})g + (1+\frac{v_e}{v_g})$$

$$\Rightarrow q = \frac{4d}{3V}\pi r^3(\rho_{oil}-\rho_{air})g + (1+\frac{v_e}{v_g})$$

I tested a variety of voltages to keep constant as V and ultimately settled on 40V because it gave me the largest range of values for velocity, and was approximately the average of the voltages I found previously using method 1.

In order to calculate the charge, I needed to find the values of the terminal velocity of the droplet under freefall, v_g and the terminal velocity in an electric field, v_e.

[7] "Viscosity of Glycerol." *Physics @ United World College Maastricht*. Web. 24 Mar. 2016.

Raw Data
Firstly, to find v_g, I used the sprayer to spray a bead of oil into the main screen when the voltage was set to 0V to find the velocity in the absence of the electric field. I did this 5 times.

Voltage = 0V

Trial	Duration ($\pm$0.01 s)	Distance ($\pm$0.001 mm)
1	0.90	0.798
2	0.69	0.609
3	0.89	0.798
4	0.91	0.807
5	0.61	0.536

Processed Data
Finding v_g:
Using a spreadsheet, I used the equation $v = \frac{distance}{time}$ to calculate the terminal velocity of a droplet in freefall. The uncertainty in the distance and duration were not included in the calculation because the relative uncertainty is quite miniscule.

Voltage = 0V

Trial	Duration ($\pm$0.01 s)	Distance ($\pm$0.001 mm)	Velocity (mm s^{-1})
1	0.90	0.798	0.89
2	0.69	0.609	0.88
3	0.89	0.798	0.90
4	0.91	0.807	0.89
5	0.61	0.536	0.88

Average v_g = 0.89 mm s^{-1}

Raw Data
After this, I needed to find the v_e in an electric field of 40V. I set the voltage to 40V, then sprayed a bead of oil into the main screen until I encountered one that rose. I started and stopped the stopwatch during the rise of the droplet and measured the distance travelled in that amount of time. I repeated this for 15 trials.

Voltage = 40V

Trial	Duration ($\pm$0.01 s)	Distance ($\pm$0.001 mm)
1	1.54	0.741
2	2.99	1.451
3	0.93	0.768
4	1.16	0.959
5	1.74	1.459
6	0.71	0.831
7	0.67	1.016
8	0.87	1.322
9	0.74	1.154
10	0.57	1.061
11	0.21	0.481
12	0.20	0.459
13	0.19	0.437
14	0.19	0.481
15	0.42	1.123

Processed Data

Finding v_e:

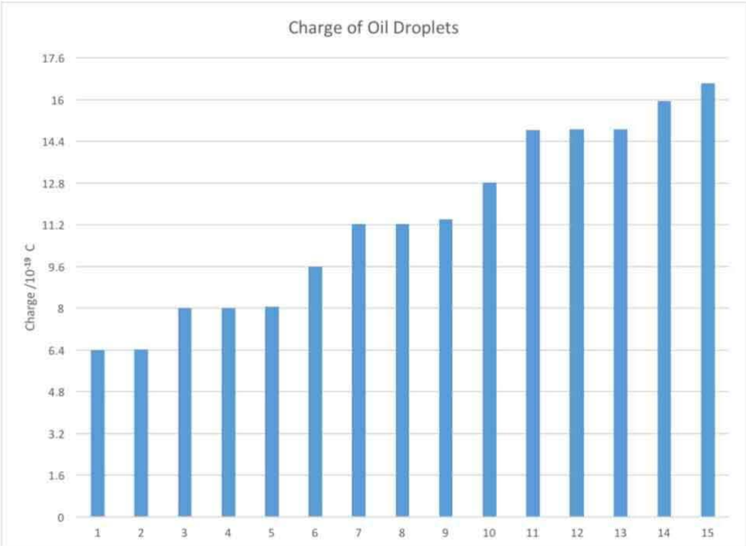
Once again, I used the equation $v = \frac{distance}{time}$ to calculate the terminal velocity of a droplet in an electric field of 40V. Like before, the uncertainty in the distance and duration were not included in the calculation.

Voltage = 40V

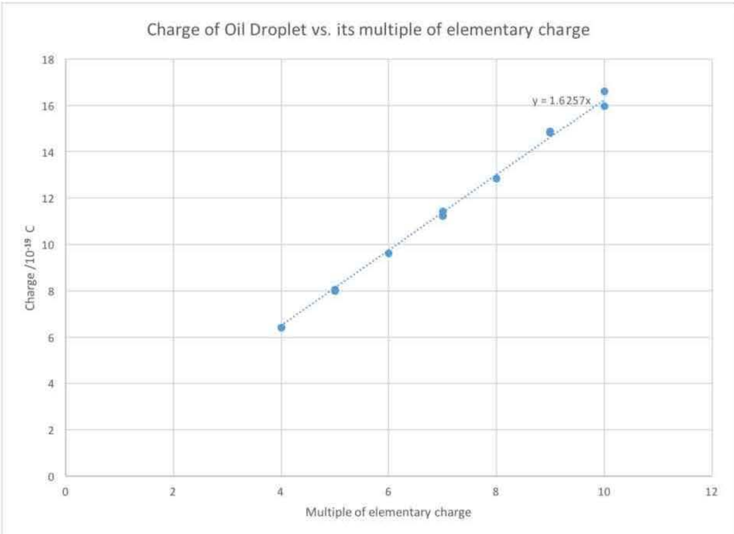
Trial	**Duration ($\pm$0.01 s)**	**Distance ($\pm$0.001 mm)**	**Velocity (mm s^{-1})**
1	1.54	0.741	0.481
2	2.99	1.451	0.485
3	0.93	0.768	0.83
4	1.16	0.959	0.827
5	1.74	1.459	0.839
6	0.71	0.831	1.2
7	0.67	1.016	1.5
8	0.87	1.322	1.5
9	0.74	1.154	1.6
10	0.57	1.061	1.9
11	0.21	0.481	2.3
12	0.20	0.459	2.3
13	0.19	0.437	2.3
14	0.19	0.481	2.5
15	0.42	1.123	2.7

To find the charges, I substituted the values I obtained into the derived equation using Excel. The values were sorted from least to greatest, then a bar graph was drawn. I then grouped the similar values into 9 clusters by their apparent multiple of elementary charge, as I did before in order to draw the line of best fit in a scatter graph.

Velocity (mm s^{-1})	**Charge (10^{-19} C)**	**Multiple of elementary charge**
0.481	6.39	4
0.485	6.41	4
0.83	8.0	5
0.827	8.00	5
0.839	8.06	5
1.2	9.6	6
1.5	11	7
1.5	11	7
1.6	11	7
1.9	13	8
2.3	15	9
2.3	15	9
2.3	15	9
2.5	16	10
2.7	17	10



Once again, there is a step pattern that seems to rise every 1.6×10^{-19} C, however it does not result in the lowest possible multiple, which shows that this method of finding the elementary charge is flawed. If I had performed this method, without first doing the other way, I may not have been able to find the smallest indivisible charge, which is the elementary charge. Nevertheless, I attempted to find the elementary charge using the gradient of the line of best fit.



As seen on the graph above, the elementary charge is approximately 1.6257×10^{-19}C.

Limitations

Although this was a computer simulation, there are a few limitations that could have affected the accuracy or precision of this investigation.

Method 1:
The microscope of the simulation had a maximum magnification of 32x, and although at this magnification, the oil drop was not rising or falling, perhaps at an even closer scale, I could have seen that it was moving slightly. Errors may have arisen from the limited ability to adjust the voltage to only 2 decimal places, and a more precise scale would have increased the precision of the data. Furthermore, the calculations could be more precise if more significant figures were given for the density of oil and air, the radius, etc.

Method 2:
An issue in the design of this method is the inability to find the smallest, irreducible value for the elementary charge. Thus, while the results of this experiment demonstrates the quantization of energy, the charge of the electron provided by this method is not as convincing as method 1. Perhaps, the voltage I used was not optimal to find the elementary charge. With more time, I can try this method again, but with different voltages, to see which ones would provide more accurate results. Reaction time and errors in measurement are an issue because although the program was highly sophisticated and marked the position that the oil drop at the start and stop of the stopwatch, I may have clicked start and stop too early and the droplet may not have reached terminal velocity. This will impact the accuracy of the experiment, especially for the droplets that rose too quickly for my reaction. Like in method 1, a greater amount of significant figures can enhance the precision of this experiment.

Conclusion
This was performed by a computer simulation, so the purpose of this investigation is merely an academic exercise to help me understand the origin of the discovery of the charge of the microscopic and enigmatic electron. In recreating the famous Millikan oil-drop experiment using the simulation and even using an alternative methodology, I was able to determine the value of elementary charge quite accurately.

The results I got were 1.6047×10^{-19}C and 1.6257×10^{-19}C, calculated by the voltage and terminal velocity, respectively. As I mentioned in the beginning of this investigation, the accepted value of the elementary charge (*e*) is approximately $1.602\,176\,6208\times10^{-19}$ C, with a standard uncertainty in the last two digits. The result from method 1 is only $\frac{1.6047\times10^{-19}-1.602\,176\,6208\times10^{-19}}{1.602\,176\,6208\times10^{-19}}\times 100\% \approx 0.12\%$ off from the accepted value and the result from method 2 is $\frac{1.6257\times10^{-19}-1.602\,176\,6208\times10^{-19}}{1.602\,176\,6208\times10^{-19}}\times 100\% \approx 1.44\%$ off.

In the future, with the proper equipment, I would like to physically perform this experiment. The computer simulation was a sophisticated program, but the values were already programmed to achieve certain results. If I could try a hands-on experiment, I can fully appreciate the wonder of Millikan's ground-breaking oil-drop experiment and see the true effect of the electric field on the oil drop. Further, I would like to learn about some of the other ways that elementary charge was experimentally measured, and compare those results to the ones of the oil-drop experiment.

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Simulations

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