# Research Question: How does the density of water affect single-slit diffraction patterns of waves?

**Introduction:**

In this investigation I will aim to find out the relationship between density and single-slit diffraction. Single-slit diffraction occurs when a wave passes through a small opening relative to the size of the wave. My interest in diffraction sparked from my passion for photography, as it relates to the circular diffraction of light waves ("Circular Aperture Diffraction"). Photography has been fundamental aspect of my life, as my father has always been a passionate photographer, capturing so many significant events throughout my life. He inspired me, and I quickly developed a similar interest that has now turned into a personal hobby. Currently I have my own camera equipment and have done jobs for people in search of business photos or at particular events. A photograph can be manipulated in various ways, one being its depth of field, dependent on the aperture you shoot on.

Aperture, shutter speed and ISO all determine the amount of light that is taken in by the camera, more specifically a DSLR. Through research, I discovered how aperture is a fraction of the focal length, thus being defined by $\frac{focal\ length}{Diameter\ of\ aperture}$. This equation shows the aperture stops, also known as "f-stops", which "limits the amount of light which passes through an optical system" ("Stops, Pupils, and Aperture"). The relationship between the aperture (size of the opening) and diffraction is well-known. Therefore, this investigation was extended by looking at how density affected diffraction patterns. However, there is an absence of literature values for this investigation, thus a link cannot be made.

**$H_0$:** There is no relationship between opening size divided by the wavelength and the diffraction angle as well as diffraction and density of the solution.
**$H_A$:** There is an inverse relationship between opening size divided by the wavelength and the diffraction angle. Furthermore, diffraction decreases as the density of the solution increases.

**Scientific Reasoning:** As water waves are mechanical waves, the increase in density of water will increase the wave speed. This is because the molecules are held closer together, causing the travelling water waves to travel faster as they are more easily transmitted between particles with tight bonds. This will make the waves diffract less as they will pass the opening with a higher velocity.

**Independent variable:** The density of the water solution.
**Dependent variable:** Diffraction patterns

- Understood through relationship $\frac{\lambda\theta}{d}$ (explained later)

**Controlled variables:**

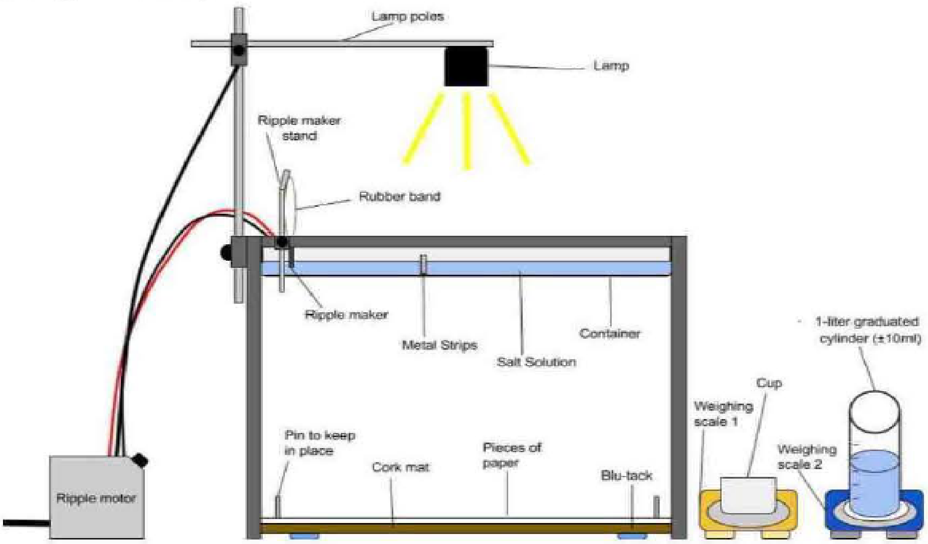
- Frequency of the wave
  - **Reason for controlling variable:** The frequency of the wave can affect how fast the wave travels. Referring to the equation $v = \lambda \times f$, we can see how a change in frequency will change the speed of the wave, which will change how much it diffracts.
  - **How it will be controlled:** This is done by the ripple motor, as the dial (kept at the same position throughout the experiment) determines the ripple maker's speed. It is assumed that the motor is not flawed to obstruct this.

- **Temperature of solution**
    - **Reason for controlling variable:** The temperature of the solution is a determinant of its density. If the temperature increases, the density of the solution will decrease causing more diffraction. If the temperature decreases, density will increase, causing less diffraction.
    - **How it will be controlled:** Measuring the temperature of the water with a thermometer for each solution. Ensuring tat the temperature is kept at room temperature (20±1°C)

**Equipment:**

- 1 Ripple tank that includes the following components:
    - Metal frame (including 4 legs)
    - 4 screws
    - Transparent container to place in the frame
    - Ripple and light motor
    - Ripple maker
        - 2 elastic bands
        - Metal holders
    - Lamp that attaches to motor
    - 2 poles for lamp stand
- 2 90° metal strips (total length of 10.3±0.1)
- 36 pieces of blank A3 paper
- Cork mat
- 4 pins
- Vernier Calliper (±0.01cm)
- Blu-tack
- Protractor of 180±1°
- 30.00±0.05cm ruler
- Weighing scale 1 (±0.01g)
- Weighing scale 2 (±0.5g)
- 1-liter graduated cylinder (±10mL)
- Measuring cup
- Laptop with Microsoft Excel
- Pen

*Fig. 1* Diagram of Setup[1]

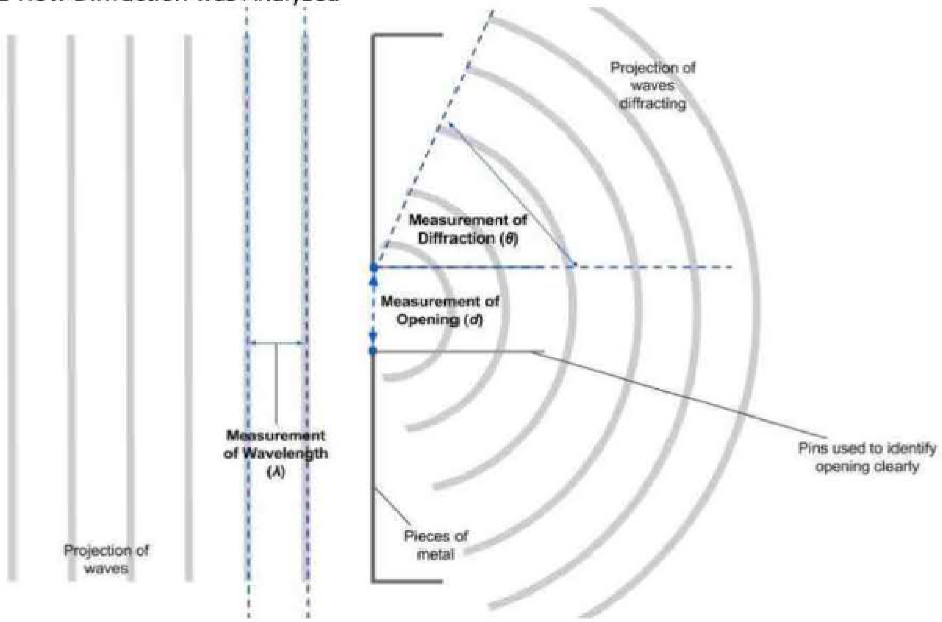



[1] Created on Google Drawings

**Methodology:**

1. Gather all the required materials necessary for the investigation
2. Set up all equipment according to the diagram above (*fig. 1*)
3. Measure mass of graduated cylinder without anything in it on weighing scale 2
4. Measure 900ml of water in the 1-litre graduated cylinder
5. Measure 50g of salt (NaCl) into the bowl on weighing scale 1, then pour it into the 900ml measured water
6. Mix until all the salt to create solution
7. Measure mass of graduated cylinder with solution in it on weighing scale 2
8. Subtract the mass of the graduated cylinder without the solution from the value recorded with the solution to find the mass of the solution alone
9. Pour solution into ripple tank container
10. Turn on ripple motor on set frequency and lamp
11. Place metal strips 5.0cm away from ripple maker and measure 0.50cm with the Vernier Calliper and adjust the opening size (distance between the two pieces of metal) to what is set by the Vernier Calliper.
12. Place pins by both ends of the opening in order to clearly identify it on the projection.
13. Identify the opening/split on the project by looking at the ends of the pins and measure the distance between the two points with a ruler.
14. Analyse diffraction of waves on A3 paper underneath the ripple tank using a ruler, protractor and pen. The procedure is illustrated on *fig. 2*. This includes:
    a. Measuring the wavelength
    b. Measuring the diffraction angle
15. Turn off ripple motor and lamp
16. Repeat steps 9-14, only changing the opening size to 1.0cm, 1.5cm, 2.0cm, 2.5cm and 3.0cm.
17. Repeat steps 4-19, however this time change the amount of salt to 0.00g, 100.00g, 150.00g and 200.00g keeping the volume of 900±10mL pf water constant throughout.
18. After all data is recorded, note all measurements in an Excel spreadsheet.
19. Process data to calculate the density of each solution by applying the equation: $\rho = \frac{m}{v}$, inserting the mass of the solution and volume in the respective places.
20. Calculate the percentage uncertainty of the different densities and apply to values.
21. Calculate the value of opening size divided by the wavelength for my x-axis values, corresponding results with the already know angles of propagation due to diffraction for my y-axis, along with their respective uncertainties.
22. Illustrate graphs showing the relationship between d/λ and θ for each individual density
23. Compare all lines of best fit on one graph to illustrate the effect of density on how much the solution diffracts
24. Conclude and evaluate

*Fig.2* How Diffraction was Analyzed[2]




The diffraction patterns were analyzed by measuring at the size of the opening, the wavelength of the wave and the diffracted angle. As the waves were projected on the white A3 paper below the ripple tank I was able to measure the various variables.

- Wavelength was easy to measure as the projections of the waves remained in the same place before the waves diffracted. Therefore, using a ruler, I simply measure the distance between two waves to find the wavelength
- The openings were clearly identified by the pins placed in the ripple tank. Measure the distance between two opening points to find *d*.
- The measurement of the diffraction was measured using a ruler from the origin line (mentioned in method). Draw another line where the waves diffracted and measure the angle in between the two lines using a protractor.

[2] Created on Google Drawings

**Analysis:**

**Density of Solutions**

To begin, at the start of each trial I calculated the density of the water by calculating the mass of the solution and the volume of the solution. From here I was able to calculate the uncertainty of the solution's density.

*Table 1.* Raw and Processed Data for Density Calculation

| Amount of Salt added to Solution ±0.01g | Mass of Solution including Graduated Cylinder ±0.5g ($m_1$) | Mass of solution ±0.5g ($m$) | Volume of solution ±10mL | Density (g/mL) | Percentage Uncertainty | Density % Uncertainty |
|---|---|---|---|---|---|---|
| 0 | 1168.0 | 900.0 | 900 | 1.000 | 0.00611111 | 0.006 |
| 50 | 1195.0 | 927.0 | 900 | 1.030 | 0.00609493 | 0.006 |
| 100 | 1219.0 | 951.0 | 900 | 1.057 | 0.00608132 | 0.006 |
| 150 | 1244.0 | 976.0 | 900 | 1.084 | 0.00606785 | 0.007 |
| 200 | 1269.0 | 1001.0 | 900 | 1.112 | 0.00605506 | 0.007 |

**Calculations:**

Firstly, I calculated the mass of the solution alone by measuring the graduated cylinder with the solution in it and from that subtracting the mass of the graduated cylinder without anything in it. The graduated cylinder alone had a mass of 268.0±0.5g.

Example:

$$m = m_1 - 268$$
$$m = 1168 - 268$$
$$m = 900 \pm 1g$$

The density was calculated using the formula: $\rho = \frac{m}{v}$. I inserted the respective values for each concentration of NaCl and calculated from there.

Example:

$$\rho = \frac{900}{900}$$
$$\rho = \mathbf{1}.000$$

The uncertainty for each calculated density value was found by first calculating the percentage uncertainty shown below:

Example:

$$\% \, uncert = \left(\left(\frac{m \, uncert \, of \, solution/2}{m \, of \, solution}\right) + \left(\frac{v \, uncert \, of \, solution/2}{v \, of \, solution}\right)\right) \times \rho \, of \, solution$$
$$\% \, uncert = \left(\left(\frac{0.5}{900}\right) + \left(\frac{5}{900}\right)\right) \times 1.000$$
$$\% \, uncert = (0.00611111) \times 1.000$$
$$\% \, uncert = 0.006$$

**Note:** *The calculations for finding the density and its uncertainty were repeated for all of the different densities.*

**Effect of Density on Diffraction Patterns**

**Density of 1.000±0.006g/mL (0g of salt added):**

*Table 2.* Raw and Processed Data for 1.000±0.006g/mL Dense Solution

| 1.000±0.006g/mL | | | | |
|---|---|---|---|---|
| **Size of split (±0.05cm) ($d$)** | **Wavelength (±0.05cm) (λ)** | **d/λ (cm)** | **% Uncertainty d/λ (cm)** | **Angle of diffraction ±1° (θ)** |
| 1.60 | 2.25 | 0.71 | 0.02 | 80 |
| 3.10 | 2.25 | 1.38 | 0.03 | 65 |
| 4.45 | 2.25 | 1.98 | 0.03 | 52 |
| 6.10 | 2.25 | 2.71 | 0.04 | 33 |
| 7.50 | 2.25 | 3.33 | 0.05 | 23 |
| 9.10 | 2.25 | 4.04 | 0.06 | 7 |

**Calculations:**

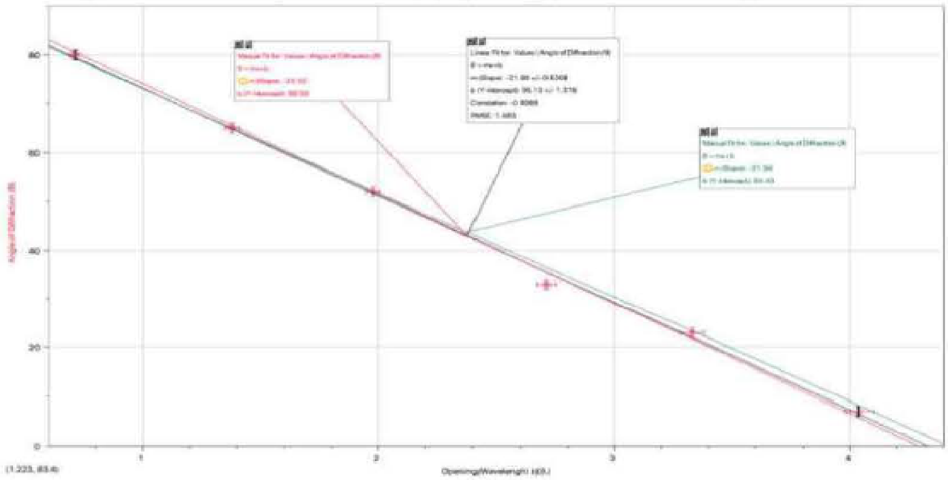
The only values that had to be calculated were those of $d/\lambda$. This was done by simply substituting the respective values into the equation. This was done for each trial for each density.

Example:

$$\frac{d}{\lambda} = \frac{2.25}{1.60} \rightarrow \frac{d}{\lambda} = 0.71$$

All the uncertainties were provided by the measuring equipment, however when calculating d/λ the percentage uncertainty had to be found for each value calculated. Example of calculating percentage uncertainties is shown above.

*Fig.3* Showing the Relationship Between (d/λ) and (θ) in 1.000±0.006g/mL Dense Solution



Above I plotted ($d/\lambda$) against θ that was calculated and measured in the 1.000±0.006g/mL dense water solution. With this I was able to plot a line of best fit alongside the maximum ($G^{max}$) (green line) and minimum ($G^{min}$) (red line) lines so that the uncertainty could be calculated.

The line of best fit of the graph has a slope of -21.96

Gradient^max ($G^{max}$) = -21.36
Gradient^min ($G^{min}$) = -22.50

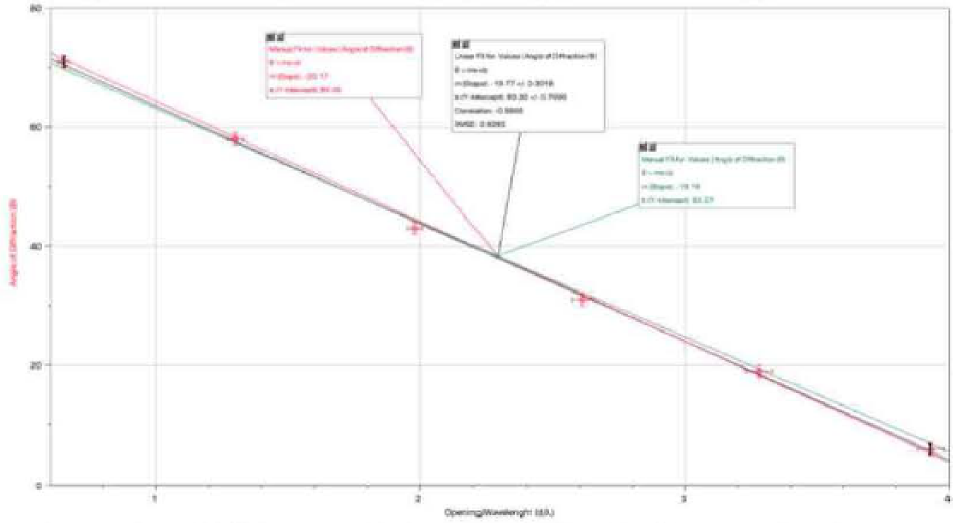
$$\frac{G^{max} - G^{min}}{2}$$
$$\frac{-21.36 + 22.50}{2} = 0.57 \approx 0.6$$

Therefore, this inverse relationship represents rate of diffraction with a value of -22.0±0.6 in a 1.000±0.006g/mL dense solution.

***Note:** All following gradient uncertainties were calculated in the same way as shown above.*

**Density of 1.030±0.006g/mL (refer to *table 3* for data):**

*Fig.4* Showing the Relationship Between (d/λ) and (θ) in 1.030±0.006g/mL Dense Solution

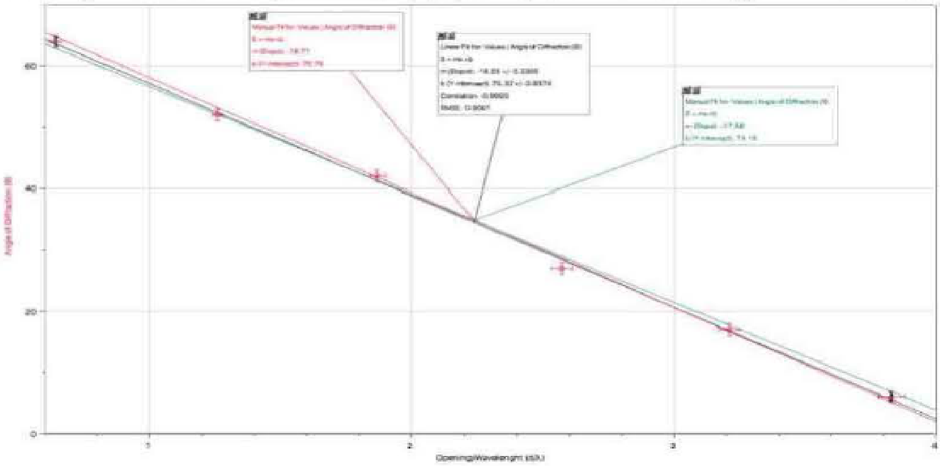


Above I plotted ($d/\lambda$) against θ that was calculated and measured in the 1.030±0.006g/mL dense water solution. With this I was able to plot a line of best fit alongside the maximum ($G^{max}$) (green line) and minimum ($G^{min}$) (red line) lines so that the uncertainty could be calculated.

Therefore, this inverse relationship represents rate of diffraction with a value of -19.8±0.5 in a 1.030±0.006g/mL dense solution.

**Density of 1.057±0.006g/mL (refer to *table 4* for data):**

*Fig.5* Showing the Relationship Between (d/λ) and (θ) in 1.057±0.006g/mL Dense Solution

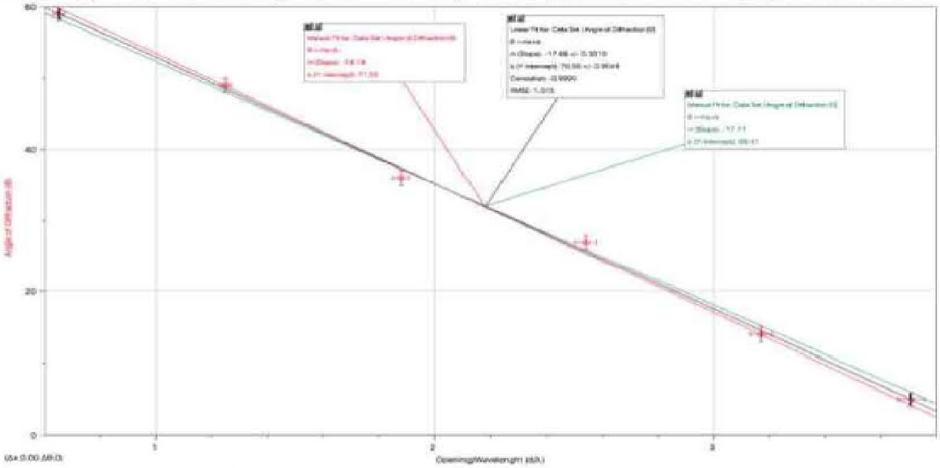



Above I plotted ($d/\lambda$) against θ that was calculated and measured in the 1.057±0.006g/mL dense water solution. With this I was able to plot a line of best fit alongside the maximum ($G^{max}$) (green line) and minimum ($G^{min}$) (red line) lines so that the uncertainty could be calculated.

Therefore, this inverse relationship represents rate of diffraction with a value of -18.4±0.6 in a 1.057±0.006g/mL dense solution.

**Density of 1.084±0.007g/mL (refer to *table 5* for data)**

*Fig.6* Showing the Relationship Between (d/λ) and (θ) in 1.084±0.007g/mL Dense Solution

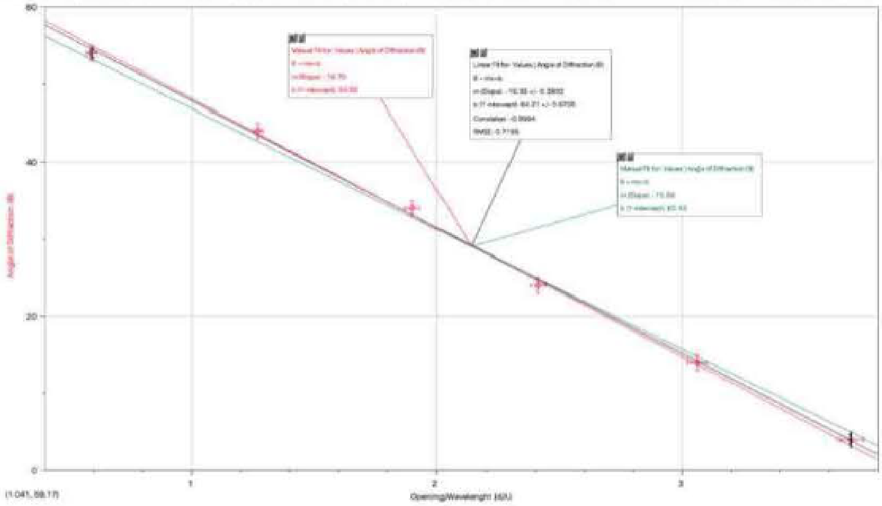



Above I plotted ($d/\lambda$) against θ that was calculated and measured in the 1.084±0.007g/mL dense water solution. With this I was able to plot a line of best fit alongside the maximum ($G^{max}$) (green line) and minimum ($G^{min}$) (red line) lines so that the uncertainty could be calculated.

Therefore, this inverse relationship represents rate of diffraction with a value of -17.7±0.5 in a 1.084±0.007g/mL dense solution.

**Density of 1.112±0.007g/mL (refer to *table 6* for data):**

*Fig.7* Showing the Relationship Between (d/λ) and (θ) in 1.112±0.007g/mL Dense Solution

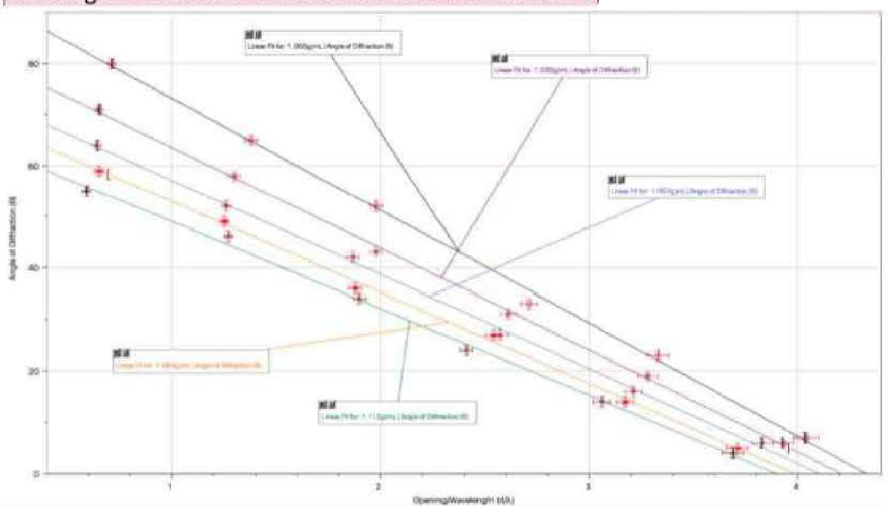


Above I plotted (*d/λ*) against θ that was calculated and measured in the 1.112±0.007g/mL dense water solution. With this I was able to plot a line of best fit alongside the maximum ($G^{max}$) (green line) and minimum ($G^{min}$) (red line) lines so that the uncertainty could be calculated.

Therefore, this inverse relationship represents rate of diffraction with a value of -16.4±0.6 in a 1.112±0.007g/mL dense solution.

**Comparing the Diffraction Patterns of Different Densities:**

*Fig. 8* Showing the Rate of Diffraction for Different Densities



Above I plotted ($d/\lambda$) against θ, where lines represent the diffraction of each density is illustrated. The black line represents diffraction of 1.000±0.06g/mL, the purple line for 1.030±0.06g/mL, the blue line for 1.057±0.06g/mL, the orange line for 1.084±0.07 and finally the green line for 1.112±0.07g/mL. From this diagram the effect of density is made clear as we can see as the density of the solution increase, it diffracts less. That being said, we can see that as density increases the distance between each line decreases, which is because the rate of diffraction gets closer to zero (becoming less steep). This is due to the increase in wavelength (which can be seen in appendix), causing the velocity to increase accordingly (based on the equation: $v = f \times \lambda$).

**Conclusion:**

From the data and referring back to the initial research question: "how does the density affect single-slit diffraction patterns of waves?", one can conclude that there is an obvious effect of density of a solution on how much it diffracts. More specifically, the data supports the alternate hypothesis, as there was an inverse relationship between the opening size divided by the wavelength and the diffraction angle, and diffraction decreased as the density of the solution. This can clearly be seen on figure 8 as both of these relationships are illustrated. This is due to the increase in velocity understood through our understanding of mechanical waves (explained in introduction) as well as $v = f \times \lambda$. As frequency remains constant, wavelength increases as density increases, increasing the velocity of the wave as the density increases. This however does contradict a literature investigation stating "the denser a solution, the smaller the wavelength" (McCowen).

Relating this back to the real-life situation of photography, I now have a greater understanding of diffraction patterns that corresponds with my understanding of aperture. As explained above, the aperture is a fraction of the focal length, which is why thee is an inverse relationship presented above. Through the investigation I have no acquired a deeper knowledge of the depth of field of a picture, or in simpler terms, focused or blurred picture. To explain, when there is a larger opening (smaller aperture), light waves diffract less, meaning that all the light will focus on a specific point of focus of the photograph, whereas the rest will be blurred, resulting in a lower depth of field. On the other hand, when the opening is smaller (larger aperture), light waves diffract more, meaning that it will disperse and reach multiple part of the image, making them more focused.

**Evaluation:**

Looking at the data and its insignificant uncertainty, one can assume that little random errors were made throughout the experiment. That being said, there are several systematic and procedural errors that could have affected the data such as:

- The density of the solution: This was determined by the amount of salt that was poured into the water to create the solution. Towards the end of the experiment, time ran out and therefore some parts of the procedure were rushed. Often when the solution was poured into the ripple tank container, some of the salt had not yet dissolved. This would ultimately effect the density of the solution as well as how much it diffracted. Although this would not have altered the experiments outcome, as the inverse relationship was still identified, it would have effected its precision. This could be avoided by letting the solution dissolve for several hours and then returning and continuing the experiment.

- The projection of the waves: This was what all the data was based on. The projections were faint and hard to clearly identify. The fringes of diffraction made it extremely difficult to identify where the angle of diffraction had to be drawn. Overall, all these factors would have affected the data in terms of its precision, however the experiment remained accurate. This could be improved by doing the experiment in a darker room that was not influenced by other light sources or by doing multiple trials for each density.
- Scaling factors: The data were measured on an image, not the actual waves. Therefore, all the values that were recorded are not representative of the actual waves created in the ripple tank. This means that the values recorded and calculated would have been different. Also, the scaling factors made the objects much larger than they actually, further influencing the precision of the experiment and the uncertainty. This could be improved by using a completely different arrangement of equipment that could measure the variables in the water.
- The amount of trials: The experiment had insignificant uncertainty as it was purely based on the equipment used. By increasing the amount of trials, more data would be able to justify the outcome of the experiment. This would also have increased the precision of the experiment as a more accurate uncertainty could be formed.
- The use of water waves: Although water waves did illustrate diffraction accurately, this did not correspond with my personal connection. Despite the same principles applying, this could have been different by looking at the diffraction of a laser light through different opening sizes. This would immediately rule out any scaling factors as well as the lack of precision that was significant throughout the experiment.

If this experiment were to be extended, other factors of the wave could be manipulated to look at how it may affect diffraction. Also, the experiment could be furthered by investigating diffraction through different mediums rather than a water solution. Moreover, it would be interesting to investigate diffraction of sound waves or light waves (mentioned above), and seeing if there is any difference compared to water waves. This could specifically investigate Fraunhofer's single-slit diffraction equation to further my knowledge of the effect of diffraction and its relevance to photography.

**Works Cited and Consulted**

"Circular Aperture Diffraction." *HyperPhysics*. Georgia State University, n.d. Web. 16 Mar. 2016. <http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/cirapp.html>.

McCowen, Erin J. "Bioconvection: The Effect of Density on Wavelength." (2008): n. pag. *Physics*. Reed College. Web. 16 Mar. 2016. <http://www.reed.edu/physics/faculty/illing/campus/pdf/bioconvection.pdf>.

"Stops, Pupils, and Aperture." *HyperPhysics*. Georgia State University, n.d. Web. 16 Mar. 2016. <http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/stop.html#c3>.

"The Wave Equation." *The Physics Classroom*. N.p., n.d. Web. 16 Mar. 2016. <http://www.physicsclassroom.com/class/waves/Lesson-2/The-Wave-Equation>.

**Appendix:**

*Table 3.* Raw and Processed Data for 1.030±0.006g/mL Dense Solution

| Size of split (±0.05cm) (*d*) | Wavelength (±0.05cm) (λ) | d/λ (cm) | % Uncertainty d/λ (cm) | Angle of diffraction ±1° (θ) |
|---|---|---|---|---|
| 1.50 | 2.3 | 0.65 | 0.02 | 71 |
| 3.00 | 2.3 | 1.30 | 0.03 | 58 |
| 4.55 | 2.3 | 1.98 | 0.03 | 43 |
| 6.00 | 2.3 | 2.61 | 0.04 | 31 |
| 7.55 | 2.3 | 3.28 | 0.05 | 19 |
| 9.05 | 2.3 | 3.93 | 0.05 | 6 |

*Table 4.* Raw and Processed Data for 1.057±0.006g/mL Dense Solution

| Size of split (±0.05cm) (*d*) | Wavelength (±0.05cm) (λ) | d/λ (cm) | % Uncertainty d/λ (cm) | Angle of diffraction ±1° (θ) |
|---|---|---|---|---|
| 1.50 | 2.35 | 0.64 | 0.02 | 64 |
| 2.95 | 2.35 | 1.26 | 0.02 | 52 |
| 4.40 | 2.35 | 1.87 | 0.03 | 42 |
| 6.05 | 2.35 | 2.57 | 0.04 | 27 |
| 7.55 | 2.35 | 3.21 | 0.04 | 16 |
| 9.00 | 2.35 | 3.83 | 0.05 | 6 |

*Table 5.* Raw and Processed Data for 1.084±0.007g/mL Dense Solution

| Size of split (±0.05cm) (*d*) | Wavelength (±0.05cm) (λ) | d/λ (cm) | % Uncertainty d/λ (cm) | Angle of diffraction ±1° (θ) |
|---|---|---|---|---|
| 1.55 | 2.4 | 0.65 | 0.02 | 59 |
| 3.00 | 2.4 | 1.25 | 0.02 | 49 |
| 4.50 | 2.4 | 1.88 | 0.03 | 36 |
| 6.10 | 2.4 | 2.54 | 0.04 | 27 |
| 7.60 | 2.4 | 3.17 | 0.04 | 14 |
| 8.90 | 2.4 | 3.71 | 0.05 | 5 |

*Table 6.* Raw and Processed Data for 1.112±0.007g/mL Dense Solution

| Size of split (±0.05cm) (*d*) | Wavelength (±0.05cm) (λ) | d/λ (cm) | % Uncertainty d/λ (cm) | Angle of diffraction ±1° (θ) |
|---|---|---|---|---|
| 1.45 | 2.45 | 0.59 | 0.02 | 55 |
| 3.10 | 2.45 | 1.27 | 0.02 | 46 |
| 4.65 | 2.45 | 1.90 | 0.03 | 34 |
| 5.90 | 2.45 | 2.41 | 0.03 | 24 |
| 7.50 | 2.45 | 3.06 | 0.04 | 14 |
| 9.05 | 2.45 | 3.69 | 0.05 | 4 |