AN INVESTIGATION MEASURING THE PERMEABILITY OF FREE SPACE CONSTANT USING SIMPLE SOLENOIDS AND A MAGNETIC FIELD PROBE

Internal Assessment

Physics

Contents

Purpose

The purpose of this investigation is to see whether the permeability of free space constant is the same in practice as it is in theory. I chose to investigate this because magnetic fields and currents have fascinated me from a young age. In first grade I was already creating my own circuits and I wanted to explore whether the practical calculations are as precise as the theoretical calculations. Furthermore, I am particularly interested in electromagnetism because of how it has changed in society and how everything is constantly being developed. It intrigues me because the whole concept of electricity is a central part of our daily lives and progression and it has changed the environment we live in.

Research question

How precise can we measure the magnetic permeability of free space constant using simple solenoids and a magnetic field probe?

Introduction

When calculating the magnetic field within a solenoid, one needs to use the magnetic permeability of free space constant

$$\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$$

$$\mu_0 = 1.2566 \times 10^{-6}\ TmA^{-1}$$

The aim of this investigation is to find out whether the constant has the same value as mentioned above in practice. This will be done by using the formula for the magnetic field in a solenoid, $\beta = \mu_0 \times \frac{NI}{\ell}$, which was found through Ampere's Law and then doing an experiment with a coil (varying the current (I)) and then measuring the magnetic field (β). In the formula, N stands for the number of turns of the solenoid and ℓ stands for the length of the solenoid. The constant for each data set can then be calculated, using $\mu_0 = \frac{\beta\ell}{NI}$, with the expectation that the values are around the same.

The units needed:

Length	Meters (m)
Turns	Number of turns (N)
Current	Amperes (Amps)
Magnetic field	Tesla (T)

Hypothesis

The hypothesis is that the values that come from the investigation will not be exactly the constant. There will be some form of error which will lead to the constant being either higher or lower than required.

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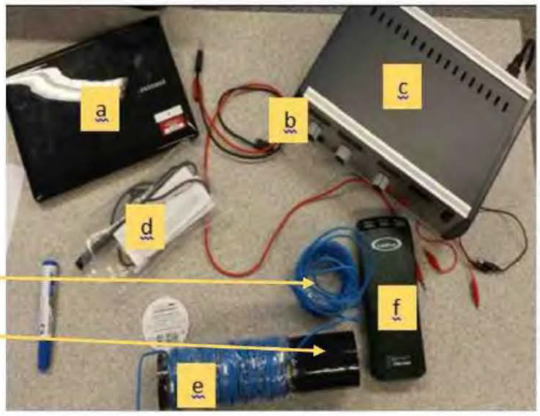
Method

Materials/apparatus

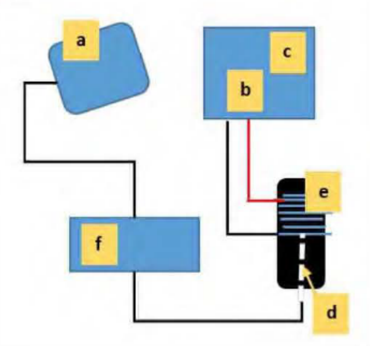
The materials needed for this investigation are:

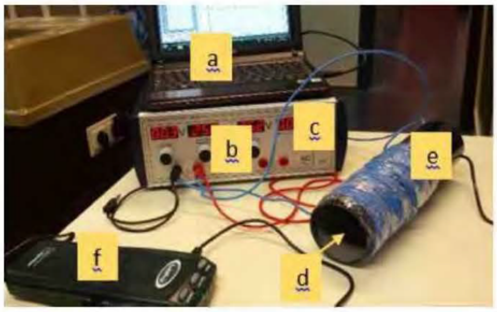
a) Laptop with Logger Pro software
b) An ammeter
c) A power source
d) A magnetic field sensor
e) A solenoid
- Insulated wire
- A hollow cylindrical shape to wrap the wire around

f) Data logger



Set-up





Variables

Independent variable: The current

Dependent variable: The magnetic field

Controlled variables: The number of turns in the solenoid = 50, 75, 100
The length of the solenoid = 21.5 cm

Risk Assessment

This experiment works with current which requires insulation in order to prevent electric shock.

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Procedure

1. Set up the experiment as shown in the diagram, beginning with a solenoid of 50 coils
2. Measure the length of the solenoid and record (ensure the length will be constant for all three solenoids)
3. Place the magnetic field sensor in the center of the solenoid, with no current going through it and record the magnetic field reading
4. Repeat step 3 ten times and then calculate an average
5. Increase the current to 0.5 Amp and record the magnetic field sensor
6. Repeat step 5 ten times and then calculate and average
7. Repeat steps 5 and 6 nineteen times, increasing the current in step five by 0.5 Amp each time
8. Remove the solenoid of 50 coils and place the solenoid of 75 coils
9. Repeat steps 3 to 7 for this solenoid
10. Remove the solenoid of 75 coils and place the solenoid of 100 coils
11. Repeat steps 3 to 7 for this solenoid

Results and Calculations

The trials provided a very large amount of data, but for this investigation only the current and corresponding average magnetic field and its standard deviation are required. First the average magnetic field needed to be calculated, then its standard deviation, and finally the magnetic permeability of free space constant could be calculated at each current.

Calculating the average magnetic field at a current of 0 Amps at 50 turns

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{0.259 + 0.261 + 0.267 + 0.255 + 0.271 + 0.263 + 0.264 + 0.259 + 0.26 + 0.239}{10}$$

$$\bar{x} = 0.260$$

Calculating the magnetic permeability constant at a current of 0.5 Amps at 50 turns

$$\mu_0 = \frac{\beta \ell}{NI}$$

$$\mu_0 = \frac{0.44 \times 0.001 \times 0.205}{50 \times 0.5}$$

$$\mu_0 = 3.60472 \times 10^{-6}$$

Calculating the standard deviation of the magnetic field of 0 Amps at 50 turns

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

$$(x_1 - 0.26)^2 = 6.4 \times 10^{-7}$$
$$(x_2 - 0.26)^2 = 1.44 \times 10^{-6}$$
$$(x_3 - 0.26)^2 = 5.184 \times 10^{-5}$$
$$(x_4 - 0.26)^2 = 2.304 \times 10^{-5}$$
$$(x_5 - 0.26)^2 = 1.2544 \times 10^{-4}$$
$$(x_6 - 0.26)^2 = 1.024 \times 10^{-5}$$
$$(x_7 - 0.26)^2 = 1.764 \times 10^{-5}$$
$$(x_8 - 0.26)^2 = 6.4 \times 10^{-7}$$
$$(x_9 - 0.26)^2 = 4 \times 10^{-8}$$
$$(x_{10} - 0.26)^2 = 4.3264 \times 10^{-4}$$

$$\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = 6.636 \times 10^{-5}$$

$$\sqrt{6.636 \times 10^{-5}} = 0.0081$$

Table 1 - Example calculations

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Solenoid of 50 turns

Current (Amps)	Average magnetic field (mT)	Standard Deviation	μ_o
0	0.260	0.0081	X
0.5	0.440	0.0217	3.60472×10^{-6}
1	0.497	0.0477	2.03606×10^{-6}
1.5	0.682	0.0134	1.86359×10^{-6}
2	0.845	0.0049	1.73266×10^{-6}
2.5	1.005	0.0125	1.64804×10^{-6}
3	1.168	0.0107	1.59654×10^{-6}
3.5	1.330	0.0128	1.55753×10^{-6}
4	1.318	0.3882	1.35044×10^{-6}
4.5	1.675	0.0093	1.52566×10^{-6}
5	1.825	0.0170	1.49658×10^{-6}
5.5	1.988	0.0283	1.48181×10^{-6}
6	2.153	0.0148	1.47149×10^{-6}
6.5	2.302	0.0331	1.45222×10^{-6}
7	2.483	0.0192	1.45421×10^{-6}
7.5	2.640	0.0130	1.44325×10^{-6}
8	2.793	0.0211	1.43126×10^{-6}
8.5	2.971	0.0228	1.43288×10^{-6}
9	3.124	0.0196	1.42297×10^{-6}
9.5	3.295	0.0166	1.42205×10^{-6}
10	3.467	0.0229	1.42151×10^{-6}
		Average μ_o	1.64227×10^{-6}
		Standard Deviation μ_o	4.79188×10^{-7} or 0.5×10^{-6}

Table 2 - Results for a solenoid of 50 turns

Solenoid of 75 turns

Current (Amps)	Average magnetic field (mT)	Standard Deviation	μ_o
0	0.223	0.0023	X
0.5	0.445	0.0046	2.43376×10^{-6}
1	0.649	0.0030	1.77503×10^{-6}
1.5	0.868	0.0100	1.58078×10^{-6}
2	1.090	0.0064	1.48898×10^{-6}
2.5	1.294	0.0086	1.41445×10^{-6}
3	1.512	0.0058	1.37742×10^{-6}
3.5	1.738	0.0099	1.35761×10^{-6}
4	1.960	0.0076	1.33913×10^{-6}
4.5	2.180	0.0297	1.32439×10^{-6}
5	2.360	0.0630	1.29024×10^{-6}
5.5	2.605	0.0210	1.29436×10^{-6}
6	2.809	0.0218	1.27961×10^{-6}
6.5	3.005	0.0608	1.26377×10^{-6}
7	3.250	0.0712	1.2692×10^{-6}
7.5	3.465	0.0529	1.26262×10^{-6}
8	3.710	0.0861	1.26758×10^{-6}
8.5	3.903	0.0343	1.25502×10^{-6}

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9	4.117	0.0370	1.25029×10^{-6}
9.5	4.338	0.0207	1.24816×10^{-6}
10	4.535	0.0328	1.23968×10^{-6}
		Average μ。	1.4006×10^{-6}
		Standard Deviation μ。	2.70537×10^{-7} or 0.3×10^{-6}

Table 3 - Results for a solenoid of 75 turns

Solenoid of 100 turns

Current (Amps)	Average magnetic field (mT)	Standard Deviation	μ。
0	0.227	0.0043	X
0.5	0.557	0.0108	2.28288×10^{-6}
1	0.901	0.0129	1.84746×10^{-6}
1.5	1.251	0.0091	1.71011×10^{-6}
2	1.618	0.0054	1.65794×10^{-6}
2.5	1.937	0.0083	1.58826×10^{-6}
3	2.290	0.0075	1.5647×10^{-6}
3.5	2.591	0.1104	1.51759×10^{-6}
4	2.965	0.0397	1.51941×10^{-6}
4.5	3.276	0.0988	1.49249×10^{-6}
5	3.560	0.0350	1.4596×10^{-6}
5.5	3.906	0.0299	1.45584×10^{-6}
6	4.263	0.0401	1.45642×10^{-6}
6.5	4.661	0.3026	1.46991×10^{-6}
7	4.923	0.0120	1.44168×10^{-6}
7.5	5.260	0.0116	1.43765×10^{-6}
8	5.689	0.0849	1.45788×10^{-6}
8.5	5.804	0.0035	1.39974×10^{-6}
9	5.809	0.0195	1.32314×10^{-6}
9.5	5.991	0.6002	1.29286×10^{-6}
10	5.786	0.0041	1.18605×10^{-6}
		Average μ。	$1.52808\times \mathbf{10^{-6}}$
		Standard Deviation μ。	2.23374×10^{-7} or 0.2×10^{-6}

Table 4 - Results for a solenoid of 100 turns

Average μ。 (TmA^{-1})			**Total average μ。**
50	**75**	**100**	1.52365×10^{-6}
1.64227×10^{-6}	1.4006×10^{-6}	1.52808×10^{-6}	

Average μ。 (for 2-10 Amps) (TmA^{-1})			**Total average μ。**
50	**75**	**100**	1.41735×10^{-6}
1.49065×10^{-6}	1.30721×10^{-6}	1.45419×10^{-6}	

Table 5 - The average constants for each solenoid

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Graphical Representation

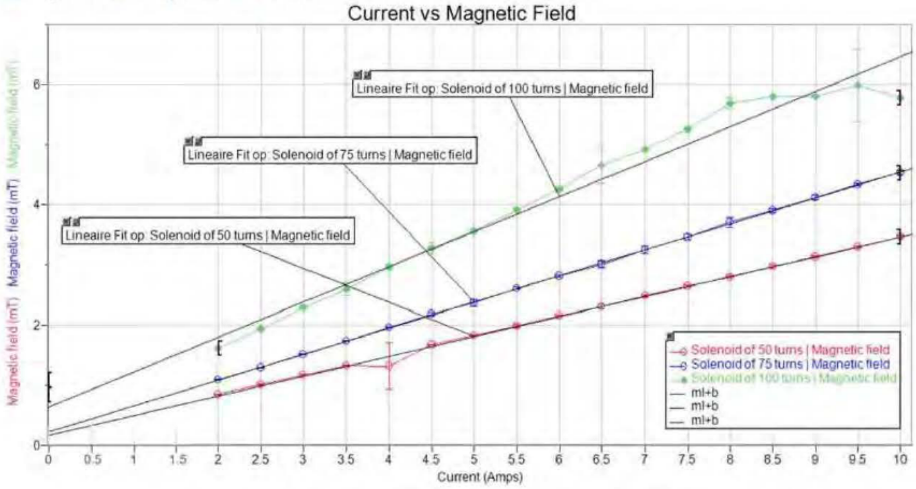

Figure 1 - The influence of current on the magnetic field

(NB. Figure 1 says "Lineaire Fit op:" which is Dutch for "the Linear fit of:")

Conclusion

In the graphical representation (figure 1) and table 6 results, the values from 0-1.5 Amps were purposefully left out. This was done because, as can be seen in tables 2-4, when there was no current going through the circuit, the magnetic field reader still recorded a background magnetic field and the Earth's magnetic field. With the readings from 2 Amps onward this background magnetic field was also picked up but it became of lesser importance and influence. The results also clearly show that from 0-1.5 Amps the value of the constant was higher than in all other values and therefore miss measured. These results were therefore removed from the calculations because it would make the final answer more accurate and closer to the required value, as can be seen in table 5.

As figure 1 illustrates, there is a linear relationship between the current (from 2-10 Amps) and the magnetic field of a solenoid of any magnitude, 50, 75, and 100 turns in this case. It can be seen that the magnetic field and current are linearly proportional to each other because the line is generally straight (especially the black lines of best fit), and (almost, with a slight increase) go through the origin. This is true for the solenoid of 50 and 75 turns and was to be expected because the formula for the magnetic permeability of free space contains the μ_0 constant. The line of best fit of the solenoid of 100 turns does not cross the origin. This is due to an anomaly, illustrated by the graph, that the magnetic field reading for the solenoid of 100 turns started to become constant after 7.5 Amps. This made the overall gradient of the line of best fit slightly to shallow, inhibiting it from crossing the origin.

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Average μ∘ (for 2-10 Amps) (TmA^{-1})			Total average μ∘	Actual μ∘
50	**75**	**100**	1.42×10^{-6}	1.2566×10^{-6}
1.49×10^{-6}	1.31×10^{-6}	1.45×10^{-6}	**Average s.d. μ∘**	
			$\pm 0.3\times 10^{-6}$	

Table 6- the average constant found

Through my investigation the average constant was calculated to be $1.42 \times 10^{-6}\ TmA^{-1}$ for the values of 2-10 Amps. The upper range of the results was $1.49 \times 10^{-6} TmA^{-1}$ and the lower range was $1.31 \times 10^{-6} TmA^{-1}$. The results may not be very precise but within the standard deviation of the average μ∘ collected, the actual value of μ∘ is obtained. The result was $0.16 \times 10^{-6} TmA^{-1}$ too large but because this is such a small value, in the micro units, this slight error can be granted. Overall the investigation was therefore successful at verifying the magnetic permeability of free space constant when calculated using simple solenoids and a magnetic field probe.

Evaluation

All in all the investigation was quite successful at measuring the magnetic permeability of free space constant using simple solenoids and a magnetic field probe. However, the investigation could have been improved in order to make the results even more accurate. To bring forth such improvements the most obvious change would be to increase both the number of trials per solenoid to 20 and the amount of solenoids tested. This would then require also solenoids of 125, 150, 175, 200 turns at least in order to really improve the average quantities found. This would be effective because, as can already be seen in the results in tables 2-4, the standard deviation continually decreases as the number of turns of the solenoid increases.

While doing the experiment itself, a couple of control variables were not completely able to be controlled. The main point was that the current would still fluctuate slightly by 0.05 Amps and the improvement to this would be to use a power source of which the current could be adjusted at a higher precision.

This experiment could also be improved by using a magnetic field senor and software that would record the magnetic field instead of simply read and display it. This is because I had to manually note down all the magnetic field but it would fluctuate so rapidly that it was near impossible to get the exact magnetic field reading. These fluctuations should not have happened because the current was fairly constant and the magnetic field sensor was held completely still.

Another issue with the magnetic field sensor used was that it measured a magnetic field when there was not current going through the circuit and measured a magnetic field that was too high for the currents of 0.5 Amps, 1 Amp and 1.5 Amps. These values were therefore omitted from the calculations making the overall results more accurate. The magnetic field reader miss measured these because it not only picked up the magnetic field

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in the solenoid, but also picked up the surrounding magnetic field of the other electric apparatus that was used as well as the Earth's magnetic field. These were also picked up with the higher currents, but were less influential because the magnetic field probe was able to focus on only the magnetic field inside the solenoid as it became stronger. Another reason that the magnetic field reader miss measured was also that the apparatus itself was simply not good enough.

In the results it can be seen that with the solenoid of 100 turns, after a current of 7.5 Amps, the magnetic field deviated from the general pattern by stopping increasing and becoming fairly constant. This was probably due to the magnetic field probe used, which perhaps could not record magnetic fields of such strengths. This should not have happened but all the results portrayed this and therefore this anomaly caused a decrease in accuracy of the overall results.

A minor occurrence noticed was that the voltage would increase slightly as the current was increased but this did not really influence the investigation itself.

Bibliography

Websites:

- Author: **HyperPhysics**, title: Ampere's Law, hosted by: Department of Physics and Astronomy of Georgia State University, publish date: unknown, accessed on: 28-11-2015 at 13:41, source: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/solenoid.html
- Author: **Department of Physics and Astronomy**, title: Magnetic field of a solenoid, hosted by: Michigan State University, publish date: unknown, accessed on: 29-11-2015 at 11:23, source: https://www.pa.msu.edu/courses/2000fall/PHY232/lectures/ampereslaw/solenoid.html
- Author: **Prof. Frank L. H. Wolfs**, title: Ampere's Law – chapter 31, hosted by: Department of Physics and Astronomy of University of Rochester, publish date: unknown, accessed on: 29-11-2015 at 12:35, source: http://teacher.pas.rochester.edu/phy122/Lecture_Notes/Chapter31/chapter31.html

Books used for general information:

- Physics Course Companion, by David Homer and Michael Bowen-Jones, published by Oxford University Press, first published in 2014

Tables:

- **Table 1 -** Example calculations, put together by the author
- **Table 2 -** Results for a solenoid of 50 turns, put together by the author
- **Table 3 -** Results for a solenoid of 75 turns, put together by the author
- **Table 4 -** Results for a solenoid of 100 turns, put together by the author
- **Table 5 -** The average constants for each solenoid, put together by the author
- **Table 6 -** the average constant found, put together by the author

Figures:

- **Figure 1 -** The influence of current on the magnetic field, created by the author using Logger Pro 3.8.6.1