I. Intro:

In high school, I began cultivating a love for stargazing. After I developed an interest in astrophysics and cosmology in my junior year, I discovered an amazing theory - one that stated that the universe is expanding. This theory, governed by Hubble's Law, states that the recessional velocity (v) of a galaxy (the velocity at which it is moving away from the earth) is directly proportional to its distance from the earth (d).

$v = H_0 d$ [1]

The constant of proportionality relating the two quantities, H_0, is known as Hubble's constant; it is usually written in the units $kms^{-1}Mpc^{-1}$. Accepted values of Hubble's constant diverge greatly because of the uncertainty involved in calculating for it, as measurements of stellar quantities such as recessional velocity and distance must be undertaken through indirect means.

After Hubble's Law was discovered, scientists also began to theorize about the forces involved in the universe's expansion. It was previously thought that the force of gravity was the primary force acting at the astronomical scale; however, the force of gravity was an attractive force and could not explain the repulsive nature of the behavior shown by Hubble's Law. Initially, the expansion of the universe was attributed solely to the momentum provided to it by the Big Bang at the beginning of its existence, but recently, scientists have suggested that the expansion of the universe may in fact also be due to dark energy, which is hypothesized to generate a repulsive force which contributes to this expansion.[2]

The expanding universe was compelling to me because it meant that the stars I was seeing in the night sky now might only be there for a short while longer on the cosmological scale - the further along the universe goes in time, the further away stars move, such that the night sky we see today is endlessly different from the one tomorrow will bring. And the expanding universe sparked a deep interest in me to wonder at the future of our universe, and the end of our known world.

Part of the reason for my wonder is the speculation that's inevitably part of Hubble's Law. Estimates for the Hubble constant of proportionality have diverged wildly over time. Even the idea that the relationship between recessional velocity and distance may be questionable, due to the large uncertainties involved in the measurements and new discoveries such as dark energy. Given this uncertainty, I am devoting this physics IA to investigating possible alternatives to the mathematical model provided by Hubble's Law, and their consequences for the future of our universe.

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[1] http://hyperphysics.phy-astr.gsu.edu/hbase/astro/hubble.html
[2] http://science.nasa.gov/astrophysics/focus-areas/what-is-dark-energy/

II. Research Question, Overview, and Variables

Research Question: What is the best mathematical model we can use to describe the expansion of the universe, and what are the implications of using this model to explain the future of our universe's development?

Hypothesis: My initial hypothesis is that an increasing exponential model will be the best mathematical model to explain the expanding universe. My reasoning for this prediction is that as galaxies move further away from the earth, they also move further away from other galaxies, and as such the gravitational force acting upon them by other stellar objects weakens as the universe expands. This weakening gravitational force means that they would experience less gravitational attraction, resulting in more rapid recessional velocities with greater distances from the earth.

Independent variable: Distance of galaxy from the earth (in Megaparsecs)

The range of distances from the earth I am considering will be from 0 Megaparsecs to 45 Megaparsecs. I am using a limited range because less data is available for stellar objects at distances further than this due to difficulties in measurement.

Dependent variable: Recessional velocity (in kms^{-1})

The reason for these units is that Hubble's constant is usually expressed in scientific literature in the unit $kms^{-1}Mpc^{-1}$.

Controlled variables:

1. Galaxy Type

The main criteria I use to select my galaxies is their similar **Hubble type** - all the galaxies I choose are spiral galaxies. The reason for keeping the Hubble type constant is that many of the indirect methods used to calculate for recessional velocity and distance are reliant on the type of galaxy involved. For example, the "standard ruler" assumption, which states that galaxies of the same Hubble type have similar actual size, is often used to calculate for the distances of galaxies. As such, to reduce the random error that would arise from analysing various types of galaxies, I have decided to select only spiral galaxies for my IA.

2. Similar galaxy spectral patterns

In addition to keeping the galaxy type constant to minimize random error in terms of distance calculations used for those galaxies, I also select galaxies with similar spectral patterns. The emission and absorption spectra of galaxies are used to calculate their recessional velocities. As such, in my IA, I selected galaxies with similar spectral characteristics. In order to control this variable, I used data provided by a Washington

University Lab to observe the spectral patterns of these galaxies and filtered out galaxies without these characteristics accordingly.

More precisely, since I am constraining my study to spiral galaxies, I search for galaxies with spectral patterns that correspond to this Hubble type. According to the Washington University Lab, I can expect to detect strong emission lines of hydrogen and absorption lines of ionized calcium (Calcium H & K) in the spectrum of a spiral galaxy[3].

3. Amount of data points gathered per galaxy

After selecting the 15 spiral galaxies for observation, I gather three separate measurements for both recessional velocity and distance from the Earth per galaxy. I keep the amount of data points gathered per galaxy constant so as to limit the variation in the accuracy of my results per galaxy.

The impact of uncontrolled variables

The effect of gravitational attraction on the motion of closely separated galaxies adds a random velocity component to the motion of galaxies that cannot be explained by Hubble's Law. This additional velocity is called the *peculiar velocity* of a galaxy, and the effect of this peculiar velocity becomes greater the closer a galaxy is to the earth, because then gravitational effects have a significant impact compared to the recessional velocity caused by the universe's expansion[4]. Precisely because it is difficult to observe faraway stellar objects, it is difficult to place a precise value onto peculiar velocities, especially for nearby galaxies. As such, peculiar velocities will be a source of random error in the measured recessional velocities shown in my IA.

III. Data Collection and Processing

1. Select 15 spiral galaxies to analyse - galaxies of similar Hubble type and spectral characteristics, but with varying distances from the Earth.

To elaborate on the first step, I demonstrate the selection process I underwent for selecting the galaxies on the galaxy NGC 1357. In this selection process, I attempt to control each of the aforementioned variables.

First, I check for the Hubble type of NGC 1357 and find that it is a spiral galaxy from a California Institute of Technology astronomy lecture[5]. This can also be observed through a simple look at a photograph of NGC 1357, where one can immediately note that the shape of NGC 1357 corresponds to that of a spiral-shaped galaxy such as our own Milky Way.

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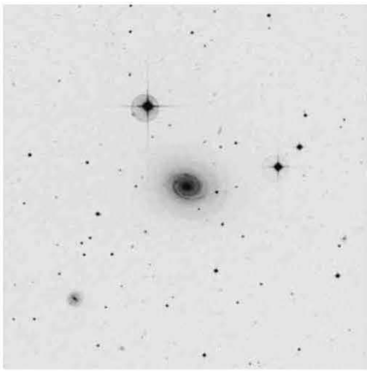
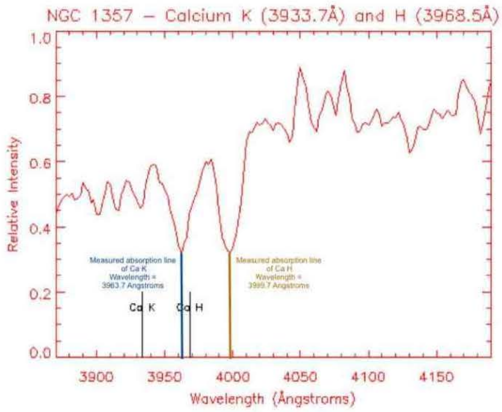
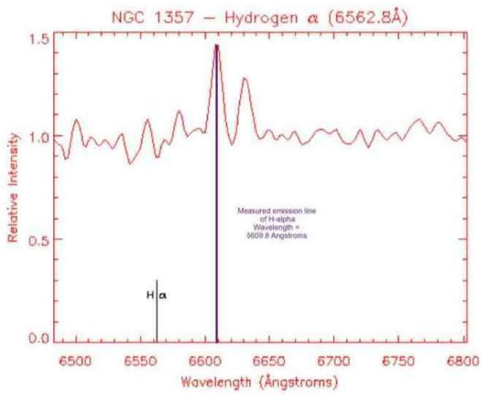
[3] http://www.astro.washington.edu/courses/labs/clearinghouse/labs/HubbleLaw/knowgalaxies.html
[4] http://astronomy.swin.edu.au/cosmos/P/Peculiar+Velocity
[5] http://www.astro.caltech.edu/ay1/lectures/2011/class15_11_short.pdf



A photograph of NGC 1357, which is the galaxy at the center

Next, I observe the spectra of NGC 1357 for the Calcium H and Calcium K absorption troughs and the Hydrogen-alpha emission peak that the Washington University lab mentioned were characteristic of the spectra of spiral galaxies.





NGC 1357 does exhibit the spectral characteristics I am searching for. The blue line shows its Calcium K absorption line, the orange line shows its Calcium H absorption line, and the purple line shows its Hydrogen-alpha emission line, while the black lines show the expected values for these spectra. Since NGC 1357 fulfills the criteria of both Hubble type and spectral characteristics that I set forth in my controlled variables, I select it as a galaxy for measurement.

The fact that the measured values for the spectral lines of NGC 1357 are of greater wavelengths than their expected values is proof that NGC 1357 is moving away from the Earth, as its spectrum is **redshifted** - exhibiting greater measured wavelengths than expected[6]. This redshift is due to the **Doppler effect** - as a light source moves away from an observer, the wavelength of that light source appears larger[7].

2. Find the recessional velocities and distances of the selected galaxies.

Once again, I demonstrate the process I underwent for data collection using the case of NGC 1357. In this step, I collect three data points each for recessional velocity and distance per galaxy using online sources and catalogues. In the case of NGC 1357, I find the following data points for distance and recessional velocity:

Distance: 26.98 Mpc, 28 Mpc, 32 Mpc
Recessional velocity: 1965 kms^{-1}, 2009kms^{-1}, 2095kms^{-1}

3. Get averages and uncertainties for the collected values.

Average distance = (26.98 + 28 + 32)/3 = 29 (2 s.f.)
Average r. velocity = (1965 + 2009 + 2095)/3 = 2023 (4 s.f.)
Distance uncertainty = (32 - 26.98)/2 = 3 (1 s.f.)
R. velocity uncertainty = (2095 - 1965)/2 = 70 (1 s.f.)

Repeating these calculations for all the other spiral galaxies I selected, I find the resulting data table.

(My final list of 15 galaxies is: NGC 1357, NGC 1832, NGC 2776, NGC 2775, NGC 2903, NGC 3034, NGC 3147, NGC 3227, NGC 3368, NGC 3623, NGC 3627, NGC 4775, NGC 6181, NGC 6643, NGC 6764. All of these galaxies fulfill the criteria of similar Hubble types and spectral patterns.)

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[6] http://coolcosmos.ipac.caltech.edu/cosmic_classroom/cosmic_reference/redshift.html
[7] http://csep10.phys.utk.edu/astr162/lect/light/doppler.html

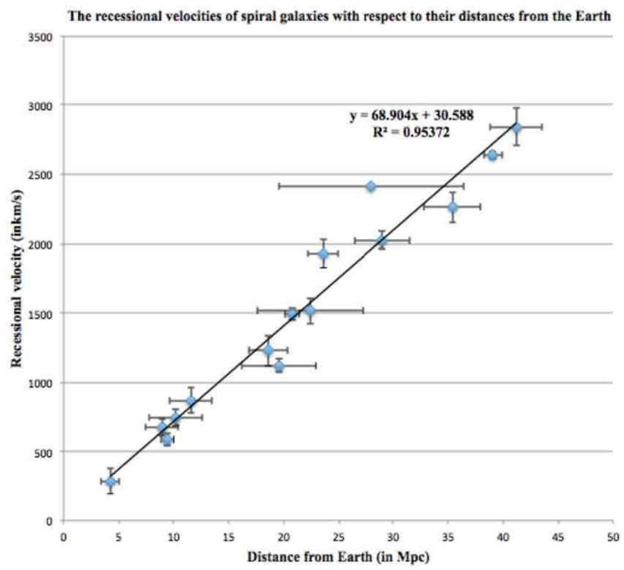
The distances and recessional velocities of fifteen selected galaxies

Galaxy	d1	d2	d3	d ave	d unc.	r1	r2	r3	r ave	r unc.
1357	26.98	28	32	29	3	1965	2009	2095	2020	70
1832	22.28	23.5	25	24	1	1823	1937	2029	1900	100
2776	38.3	38.9	40	39.1	0.9	2618	2626	2673	2640	30
2775	17	18.4	20.5	19	2	1135	1197	1354	1200	100
2903	8.9	9.46	10	9.5	0.6	556	565	644	590	40
3034	3.53	3.9	5.2	4.2	0.8	186	300	377	300	100
3147	39.26	40.3	44	41	2	2721	2812	2992	2800	100
3227	15.7	20.6	22.4	20	3	1059	1145	1157	1120	50
3368	10.1	10.52	14	12	2	762	899	943	870	90
3623	7.4	11	12.19	10	2	683	755	807	750	60
3627	7.34	9.1	10.3	9	1	603	706	727	680	60
4775	16.9	23.71	26.6	22	5	1396	1566	1582	1510	90
6181	33	35	38.1	35	3	2158	2253	2377	2300	100
6643	20	21	21.28	20.8	0.6	1454	1487	1538	1490	40
6764	20.61	26	37.4	28	8	2412	2415	2416	2414	2

Note: d1, d2, d3 stand for the three distance measurements, d ave for the average of the distances, and d unc. for the calculated uncertainty. Likewise r1, r2, r3 stand for the three recessional velocity measurements, r ave for the average of the recessional velocities, and r unc. for the calculated uncertainty. Distance is in Mpc, while recessional velocity is in kms^{-1}. I've omitted "NGC" from the names of the galaxies to conserve space in the table and rounded off appropriately in the average and uncertainty columns.

4. Graph recessional velocity (kms^{-1}) against distance (Mpc), and find the best fit line. The slope of this BFL gives me my value for Hubble's constant, which will confirm the relative accuracy of my gathered data.

Placing recessional velocity on the y-axis and distance on the x-axis, and using Excel to find the best fit line for the data, I find the following graph:



The equation of the best fit line for the data points is y = 68.904x + 30.588. However, this equation is rather precise given the uncertainty of the data involved; as such, I first solve for the uncertainty of the slope by solving for the maximum and minimum slope.

My value for the maximum slope = 79.41 $kms^{-1}Mpc^{-1}$
My value for the minimum slope = 58.08 $kms^{-1}Mpc^{-1}$
Uncertainty = (79.41-58.08)/2 = 10 (1 s.f.)

With this uncertainty in mind, the value for Hubble's constant I find (which is the gradient of the best fit line) is 70 ± 10 $kms^{-1}Mpc^{-1}$. This interval of error includes the measurements of 67$kms^{-1}Mpc^{-1}$ from the Planck satellite mission, 69.7$kms^{-1}Mpc^{-1}$ from the WMAP (Wilkinson Microwave Anisotropy Probe) 9 Analysis, and 74.3$kms^{-1}Mpc^{-1}$ from the Carnegie Hubble Program[8] within its threshold of uncertainty and, as such, I conclude that the plotted data points are reasonably accurate, though not very precise (my uncertainty for Hubble's constant is about 14% of the calculated value).

However, one error can be seen in how the equation of the best-fit line does not show a relationship of direct proportion - instead, there is a positive term of +30.588. This means that there was a systematic error involved in the measurements or procedure. The reason for this error is most likely a fault in the data I gathered from my sources.

[8] http://planck.caltech.edu/pub/2015results/Planck_2015_Results_XIII_Cosmological_Parameters.pdf

5. Analyse the linear model

y = 68.904x + 30.588	y' = 68.904 (estimated at 70)	y'' = 0	R^2 = 0.95372

When analysing the mathematical models in my IA, the major assumption I make is to consider that galaxies further away from the earth are also further forward in the expansion of the universe. Before analysing the different mathematical models, though, I will first review the linear model to clarify what quantities I am employing in my analysis.

Along with the equation of the line, I also analyse the first derivative (y'), the second derivative (y''), and the r-squared value or coefficient of determination (R^2). Since y' gives the value of the slope of the tangent line at any point along the graph, y' effectively tells me the rate at which the universe is expanding at any point along the graph. Meanwhile, y'' is the rate of change of y', and as such gives me information about how the slope itself changes over time. Finally, R^2 tells me how closely the equation employed fits the data points - the closer R^2 is to one, the more accurately the equation fits the data. However, R^2 has limitations as an analysis tool, as it only states how closely the trendline fits the data; it does not inform me if the mathematical model makes qualitative sense, which is why I couple the R^2 measurement with a qualitative analysis of each model.

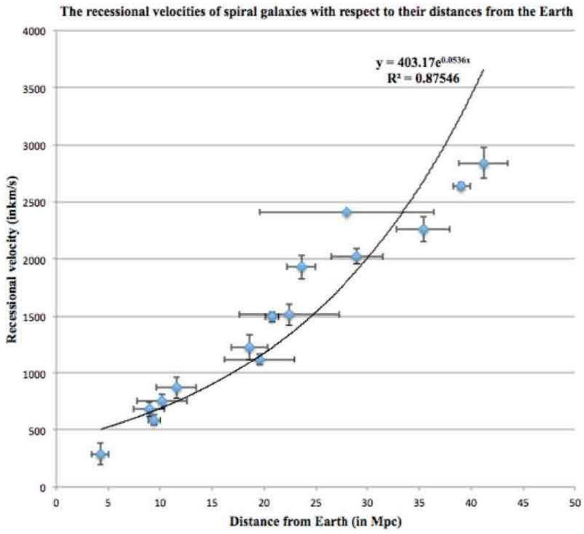
In the case of the linear model, the first derivative is a constant value and the second derivative is zero. This suggests that if the linear model were true, then the universe would expand at a constant rate, no matter the value of x (the distance of the galaxies from the earth). The R^2 value for the linear model is quite close to 1, meaning that the linear model is an accurate fit. Also, the linear model only has two outlier data points out of fifteen (I consider an outlier as any point whose interval of error does not coincide with the plotted trendline), meaning that it reasonably fits the data.

In my analysis, I also hypothesize the physical consequences of each mathematical model. In this linear model, the combined repulsive force of the universe's momentum and dark energy exceed the attractive force of gravity, but only to the extent that they allow for a constant rate of expansion. The physical consequences of the linear model being true correspond with the theory of an **open universe**[9] - one where the universe does not have enough mass for the gravitational force to outweigh the force of expansion, resulting in an infinite process of expansion.

[9] http://www2.lbl.gov/Science-Articles/Archive/SNAP-3.html

6. Graph and analyse logarithmic, exponential, and polynomial models

The exponential model:



$y = 403.17e^{0.0536x}$	$y' = 21.6099e^{0.0536x}$	$y'' = 1.15829e^{0.0536x}$	$R^2 = 0.87546$

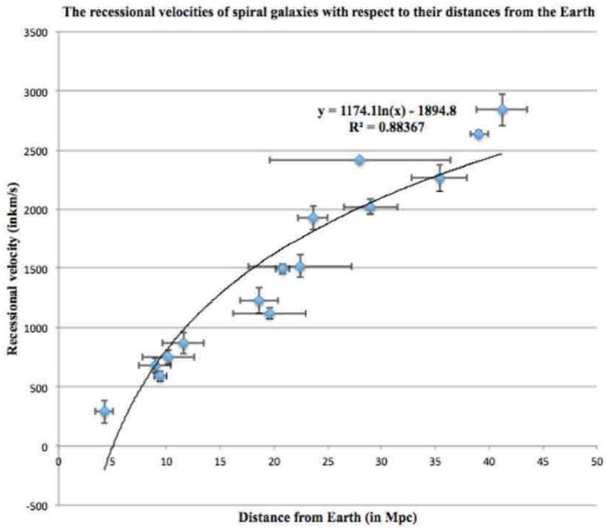
The exponential model has a horizontal asymptote at y = 0, meaning that the graph is undefined at the origin. This means that the exponential model is insufficient for explaining the trend of recessional velocity vs. distance at distances that are close to the earth. However, this does not discount the possible effectiveness of the model, because at distances close to the earth, the peculiar velocities of galaxies have significant magnitude compared to their recessional velocities, resulting in large random error. As such, the exponential model may be valid at larger distances from the earth.

The first derivative of the exponential model as x approaches infinity (or as the distances of galaxies grow greater) also approaches positive infinity, meaning that an exponential model of the expanding universe entails that galaxies are moving away from the earth at an increasing rate with respect to their distances. The second derivative of the exponential model also approaches positive infinity as x approaches infinity; in other words, the rate at which the universe expands in this model is also increasing with respect to time.

The R^2 value of the exponential model is lower than that of the linear model, meaning that it is a worse statistical fit. In addition, the exponential model has several outliers - a total of seven data points do not fit on the exponential trendline. Since this is nearly half the data points involved, this severely challenges the effectiveness of the exponential model.

The exponential model describes a universe where the repulsive force generated by the universe's momentum and dark energy far exceeds the attractive force of gravity, resulting in a universe that is forever expanding at an ever increasing rate. Similarly to the linear model, the exponential model describes an open universe, albeit one where the rate of expansion grows far more quickly.

The logarithmic model:



$y = 1174.1 \ln x - 1894.8$	$y' = 1174.1/x$	$y'' = -1174.1/x^2$	$R^2 = 0.87548$

Similarly to the exponential model, the logarithmic model is undefined close to the origin, but this time due to a vertical asymptote at x = 0. The model may still be valid for larger values of x due to the same reasons as I specified in the exponential model, but I must disregard its validity for values extremely close to x = 0 due to this asymptote.

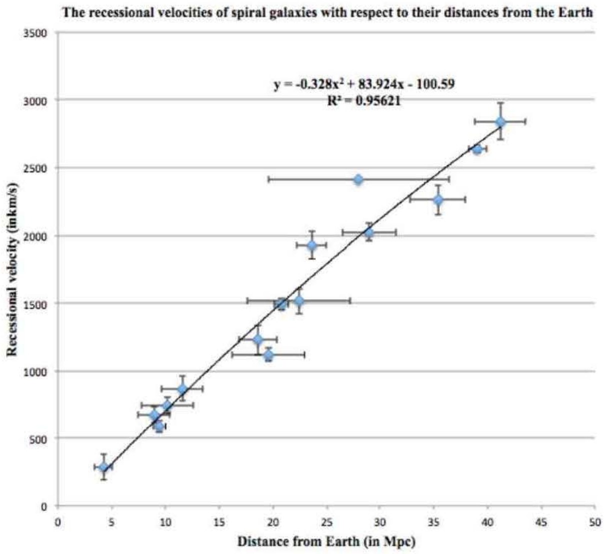
In the logarithmic model, as x approaches infinity, the first derivative remains positive, meaning that the universe is expanding in this model. However, the second derivative is negative, meaning that, in this model, the rate of the universe's expansion is actually slowing down. Taking the limit of the first derivative as x approaches infinity also shows that as galaxies move further away, y' approaches zero, meaning that at a certain point in time infinitely far away a universe following the logarithmic model would eventually stop expanding.

This is an interesting possibility for the future of our universe's expansion - it entails that perhaps the universe's initial momentum and dark energy push it to expand, but eventually that expansion comes to a halt due to gravitational attraction. The result would be that at a distance very far (infinitely far, mathematically) away from the earth, galaxies would stop moving away from one another and the universe would become static.

Statistically, the R^2 value of the logarithmic model is less than that of the linear model, meaning that it does not fit the data points as well. In addition, this model has eight outliers - more outliers than both the exponential and the linear model. This means that the logarithmic model is likely unfeasible.

Qualitatively, this model corresponds to the theory of the **flat universe**[10], which is one theory for the future of the universe's expansion. In a flat universe, there is exactly enough mass for the gravitational force to cause expansion to stop, but only after an infinite amount of time. In this model, the universe still expands forever, but this rate of expansion gradually approaches zero.

The polynomial model:



$y= -0.328x^2 + 83.924x - 100.59$	$y' = -0.656x + 83.924$	$y'' = -0.656$	$R^2 = 0.95621$

Before analysing the polynomial model shown above, I take note that systematic error also exists in this model - if x = 0, y is a negative value. I expect the graph to pass through the origin (since x=0 corresponds to the earth, obviously recessional velocity also equals zero at this point as the earth's velocity with respect to itself is always zero); this systematic error is likely due to some inaccuracies in the data points I gathered.

[10] http://www2.lbl.gov/Science-Articles/Archive/SNAP-3.html

In the polynomial model, there is a period where the universe is expanding: for certain smaller values of x, y' is positive and as such the galaxies that are further away from the earth are also moving away from it. However, after a certain distance from the earth is reached, y' becomes negative, and though expansion continues it begins to slow down. Eventually, if the graph is extrapolated for very large values of x, galaxies actually begin to **recede** and move towards the earth instead of away from it.

Interestingly, the polynomial model has an R^2 value that is slightly greater than that of Hubble's Law. The polynomial model has three outliers, a number extremely close to the linear model's two outliers, meaning that it statistically is a good fit for the data points. However, a better statistical fit does not always mean a better model, as there are qualitative limitations to consider. For example, a polynomial model might work only for the range of galaxies I chose (from 0-45 Mpc away from the earth), and might fail for galaxies at even larger distances.

The polynomial model implies a **closed**[11] universe where the universe contains enough mass for the attractive force of gravity to eventually overpower the repulsive forces behind the expanding universe. The eventual result of this would be the universe's contraction. The end of this universe would be a so-called "Big Crunch," where galaxies are once again compressed into a very tiny part of space as in the Big Bang.

[11] http://www2.lbl.gov/Science-Articles/Archive/SNAP-3.html

IV. Conclusion and Evaluation

Conclusion:

After describing the quantitative and qualitative aspects of each mathematical model, I now compare the four I've discussed and decide on which one best fits the data.

Qualitatively, all four mathematical models describe a feasible option for the future of the universe's expansion, because there is still great uncertainty in the scientific community about the development of the universe's future and whether the universe is in fact open, closed, or flat. This is because of recent discoveries which have resulted in scientists hypothesizing the existence of **dark matter**, which would add extra mass to the universe and subsequently extra gravitational force, and **dark energy**, which results in a repulsive force and thus contributes to the universe's expansion. These quantities are dark precisely because little is known about their value - no one is sure how much dark matter there is in the universe, or how much force is exerted by dark energy[12].

Recent observations of type Ia supernovae, however, show that these stellar objects are farther away from the earth than is predicted by Hubble's law. This suggests that the universe may be expanding at an accelerating rate, lending credence to the fate suggested by the exponential model[13].

Statistically, however, there are several inconsistencies present in the exponential model. Its R^2 value is the lowest among all the models; nearly half the data points do not fit the exponential trendline; the exponential model has a horizontal asymptote and thus does not make sense for all values of x. All of these errors in the exponential model lead me to reject its validity, despite its possible utility in explaining an accelerating universe. The discovery of recent observations of type Ia supernovae in my research also leads me to recognize the constraints of my model - since I limited my study to spiral galaxies, I was not able to observe the recessional velocities and distances of these supernovae. As such, my rejection of an exponential model is made with the constraints of the data I selected in mind, and does not necessarily preclude the possibility of an accelerating expansion.

For similar statistical reasons, I must also reject the logarithmic model. Its R^2 value is the second lowest among all the models; it has even more statistical outliers than the exponential model; and it similarly has an asymptote, except a vertical one.

This leaves the linear and the polynomial model as possible models for the universe's expansion. Statistically, the polynomial model has a higher R^2 value than the linear model, though it also has one more outlier; as such, these two models are comparably good fits for the data. However, by calculating for the roots of the polynomial model, one can find that it predicts that recessional velocities of galaxies will return to zero at a distance of

[12] http://science.nasa.gov/astrophysics/focus-areas/what-is-dark-energy/
[13] https://www.eso.org/~bleibund/papers/EPN/epn.html

about 255 Mpc, and then galaxies will begin moving towards the earth at distances greater than this.

This can easily be shown to be untrue through the observation of another type of stellar object - that of the quasar. Quasars are massive, extremely energetic, and very remote stellar objects - the majority of quasars are farther than 1000 Mpc away. If the polynomial model of the universe's expansion were true, one would expect these quasars to be moving towards the earth; however, observation indicates otherwise, as quasars nevertheless exhibit a large redshift, meaning that their recessional velocities are still away from the earth at these large distances[14]. Given this data, I must reject the polynomial model.

From this analysis, I conclude that out of all the models discussed, the linear model still provides the best explanation for the development of the universe. To reiterate, the linear model entails that the universe will expand at a constant rate indefinitely, describes an open universe where the repulsive nature of the momentum of the universe's expansion and dark energy outweigh the attractive force of gravity, and thus one which would expand to an infinite size.

Since the data I collected gave me a value for Hubble's constant that was close to accepted literature values, and since I sourced my measurements for distance and recessional velocity from various sources, I also note that my data is generally reliable and accurate, thus giving more credence to this conclusion.

Evaluation:

In this evaluation, I address problems with my experimental procedure and possible solutions.

1. Not enough data points

Though I initially thought 15 galaxies were sufficient for my study, more data points would have definitely been helpful in backing up the mathematical models I used. This is especially true because my analyses of the models employed involved hypothesizing about very large values of x; however, in reality, my graphs only contained data points within a finite range of distances from the earth. Though of course it would be impossible to find galaxies at infinite distances from the earth, it would have been far more effective to include more galaxies at larger distances from the earth so that the validity of the models could be seen, statistically, over a larger range.

2. Inaccuracy of online sources employed

One possible source of error in the procedure I employed was that, when taking the data points for the galaxies (data on their recessional velocities and distances from the earth), I used different online sources for different galaxies - not all galaxies had their data sourced from the same website or catalog. This procedural choice was made out of

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[14] http://csep10.phys.utk.edu/astr162/lect/active/quasars.html

necessity, because no online database contained all of the data for all of the galaxies I selected for the study. Though this was a source of error, I also wanted to access several sources to reduce error, by averaging three quantities per galaxy's measurement instead of basing all my measurements on one source. In the end, there were two options: either use one source for all my data and have limited measurements available, or use several sources for my data and have several possible measurements available. If I were to use the first method, data collection would have been easier, and there would have been less random variation among the data collected; however, if the selected source were wrong in its measurements, then that incorrectness would have been seen in all the galaxies, resulting in a notable systematic error. If I were to use the second method (the one I did end up using), then the inaccuracies of individual sources would be balanced out because of the averaging of the measurements of various sources. However, this method also introduces random error through variations in the accuracy of the measurements due to the variety of online sources employed, and the possible unreliability of using large numbers of online sources.

Neither method is perfect, so there is no perfect solution to this problem. However, two solutions I envisioned are either: a) using method one, but with a trustworthy and verified source (such as an official NASA database), or b) using method two, but with a more well-defined catalogue of sources and criteria (for example, strictly saying that each galaxy would have three measurements per quantity taken from three selected sources.)

3. Uncertainty regarding the data collection methods of online sources

When viewing online sources, it is also unclear how they procured their data for the galaxies involved. Did the online sources measure the stellar quantities, or base their sources off of scientific papers or other online sources? If they measured the stellar quantities, what techniques did they use? This variation between the techniques used for procuring data among online sources is a source of random error. Once again, there is no easy solution - if sources are limited to a certain type of data collection method, then any systematic error in that method would be found in all the sources. But if sources were not limited to a certain type of data collection method employed, then random error would abound. This is an inevitable source of unreliability in my exploration.

4. The decision to constrain the study to spiral galaxies

Though this decision was made to best control the variables for the experiment, it has its downsides. The decision to constrain measurements to a certain type of galaxy limits the effectiveness of the experiment's conclusions, as the trends concluded might not apply in the same way to other types of galaxies. The best way to resolve this issue would be to either a) repeat the study, but with different kinds of galaxies and control that variable each time, or to b) repeat the study, with several kinds of galaxies in the x-y plot all at the same time. Once again, there are pros and cons - if option a is taken, then variables can be controlled and random error is minimized, but the mathematical results would be limited to certain types of galaxies and one might not be able to make a general conclusion for all galaxies from the specific results per galaxy type. If option b is taken, then a general conclusion could be made for galaxies of various galaxy types, but then random error would be introduced in the data collection methods used for the galaxies and any characteristics peculiar to certain galaxy types that would skew the results of the experiment.

After taking a look at all the ways to improve and extend the investigation, if given more time to continue researching on this IA and to take alternate approaches, I would analyse the results given by each alternate approach and compare the most valid trends then. Would the linear model still hold true, or would the addition of different approaches give me a different conclusion? I'm also piqued by the recent observations of Type Ia supernovae by scientists, and if there were a way to conduct a similar exercise on these stellar objects, I would enjoy extending my investigation in that field as well. Though I considered switching my IA to focus on these supernovae midway through my research, I had difficulty accessing reliable data for these elusive stellar objects, so I continued with my original plan.

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