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## Objectives After finishing this unit, you should be able to:

- Write and apply relationships among pressure, volume, temperature, and quantity of matter for ideal gases undergoing changes of state.
- Define and apply concepts involving molecular mass, moles, and Avogadro's number.
- Write and apply the general gas law for a particular state of an ideal gas.


## Thermodynamic State

The thermodynamic state of a gas is defined by four coordinates:

- Absolute pressure, P
- Absolute temperature, T'
- Volume, V
- Mass $m$ or quantity of matter $n$



## Gas Laws Between States

Boyle's Law, Charles' Law, and Gay-Lusac's Law can be combined into a single formula for an ideal gas that changes from State 1 to another State 2.

| State | $\begin{array}{c}\boldsymbol{P}_{\mathbf{1}}, \boldsymbol{V}_{\boldsymbol{1}} \\ \boldsymbol{T}_{\boldsymbol{1}} \boldsymbol{m}_{\boldsymbol{1}}\end{array}$ |
| :---: | :---: |

$$
\frac{P_{1} V_{1}}{m_{1} T_{1}}=\frac{P_{2} V_{2}}{m_{2} T_{2}}
$$

$$
\begin{aligned}
& P_{2,} V_{2} \\
& T_{2} m_{2}
\end{aligned}
$$

State
2

Any Factor that remains constant divides out

Example 1: An auto tire has an gauge pressure of 28 psi in the morning at $20^{\circ} \mathrm{C}$. After driving for hours the temperature of air inside the tire is $30^{\circ} \mathrm{C}$. What will the gauge read? (Assume 1 atm $=14.7$ psi.)

$$
\begin{aligned}
& \tau_{1}=20+273=293 K \\
& \tau_{2}=30+273=303 K
\end{aligned}
$$

$$
P_{\text {abs }}=P_{\text {gauge }}+1 \text { atm; } \quad P_{1}=28+14.7=42.7 \text { psi }
$$



Same air in tires: $m_{1}=m_{2}$ Same volume of alis: $V_{1}=V_{2}$

Example 1: What will the gauge read?
Given: $T_{1}=293 \mathrm{~K} ; \quad T_{2}=303 \mathrm{~K} ; \quad P_{1}=42.7 \mathrm{psi}$ $\frac{P_{P} X_{1}}{y_{1} T_{1}}=\frac{P_{X} X_{2}}{p_{2} T_{2}}$

## $\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}}$


$P_{2}=\frac{P_{1} T_{2}}{T_{1}}=\frac{(42.7 \mathrm{psi})(303 \mathrm{~K})}{293 \mathrm{~K}}$

$$
P_{2}=44.2 \text { psij }
$$

Gauge pressure is 14.7 psi less than this value:

$$
P_{2}=44.2 \mathrm{psi}-14.7 \mathrm{psi} ;
$$

$$
P_{2}=29.5 \mathrm{psi}
$$

## The Composition of Matter

When dealing with gases, it is much more convenient to work with relative masses of atoms.

Building blocks of atoms.



Atoms contain protons and neutrons o, which are close to the same mass, surrounded by electronso which are almost negligible by comparison.

## Relative Masses

To understand relative scales, let's ignore electrons and compare atoms by total number of nuclear particles.

Hydrogen, H
Helium, He
Lithium, Li
Carbon, C
Oxygen, O ececece00000000

1 particle
4 particles
7 particles
12 particles
16 particles

## Atomic Mass

The atomic mass of an element is the mass of an atom of the element compared with the mass of an atom of carbon taken as 12 atomic mass units (u).

Atomic masses of a few elements:
Hydrogen, $\mathrm{H}=1.0 \mathrm{u}$
Helium, $\quad \mathrm{He}=4.0 \mathrm{u}$
Lithium, $\quad \mathrm{Li}=7.0 \mathrm{u}$
Beryllium, $\mathrm{Be}=9.0 \mathrm{u}$

Carbon, $\quad \mathrm{C}=12.0 \mathrm{u}$
Nitrogen, $\mathrm{N}=14.0 \mathrm{u}$
Neon, $\quad \mathrm{Ne}=20.0 \mathrm{u}$
Copper, $\mathrm{Cu}=64.0 \mathrm{u}$

## Molecular Mass

The molecular mass M is the sum of the atomic masses of all the atoms making up the molecule.

## Consider Carbon Dioxide $\left(\mathrm{CO}_{2}\right)$

The molecule has one carbon atom and two oxygen atoms

$$
\begin{array}{r}
1 \mathrm{C}=1 \times 12 \mathrm{u}=12 \mathrm{u} \\
2 \mathrm{O}=2 \times 16 \mathrm{u}=32 \mathrm{u} \\
\hline \mathrm{CO}_{2}=44 \mathrm{u}
\end{array}
$$

## Definition of a Mole

One mole is that quantity of a substance that contains the same number of particles as there are in 12 g of carbon-12. ( $6.023 \times 10^{23}$ particles)

1 mole of Carbon has a mass of 12 g
1 mole of Helium has a mass of 4 g
1 mole of Neon has a mass of 20 g
1 mole of Hydrogen $\left(\mathrm{H}_{2}\right)=1+1=2 \mathrm{~g}$
1 mole of Oxygen $\left(\mathrm{O}_{2}\right)$ is $16+16=32 \mathrm{~g}$

## Molecular Mass in grams/mole

The unit of molecular mass $M$ is grams per mole.
Hydrogen, $\mathrm{H}=1.0 \mathrm{~g} / \mathrm{mol} \mathrm{H}_{2}=2.0 \mathrm{~g} / \mathrm{mol}$
Helium, $\quad \mathrm{He}=4.0 \mathrm{~g} / \mathrm{mol} \quad \mathrm{O}_{2}=16.0 \mathrm{~g} / \mathrm{mol}$
Carbon, $\mathrm{C}=12.0 \mathrm{~g} / \mathrm{mol} \quad \mathrm{H}_{2} \mathrm{O}=18.0 \mathrm{~g} / \mathrm{mol}$
Oxygen, $\quad \mathrm{O}=16.0 \mathrm{~g} / \mathrm{mol} \quad \mathrm{CO}_{2}=44.0 \mathrm{~g} / \mathrm{mol}$
Each mole has $6.23 \times 10^{23}$ molecules

## Moles and Number of Molecules

Finding the number of moles $n$ in a given number of N molecules:

$$
n=\frac{N}{N_{A}}
$$

Example 2: How many moles of any gas will contain $20 \times 10^{23}$ molecules?

$$
n=\frac{N}{N_{A}}=\frac{20 \times 10^{23} \text { molecules }}{6.023 \times 10^{23} \text { molecules } / \mathrm{mol}}
$$

$$
\mathrm{n}=3.32 \mathrm{~mol}
$$

## Moles and Molecular Mass M

Finding the number of moles $n$ in a given mass m of a substance:

$$
n=\frac{m}{M}
$$

## Molecular mass M is expressed in grams per mole.

Exsample 3: How many moles are there in 200 g of oxygen gas $\mathrm{O}_{2}$ ? $(\mathrm{MI}=32 \mathrm{~g} / \mathrm{mol})$

$$
n=\frac{m}{M}=\frac{200 \mathrm{~g}}{32 \mathrm{~g} / \mathrm{mol}}
$$

$$
\mathrm{n}=6.25 \mathrm{~mol}
$$

Example 4: What is the mass of a single atrom of boron ( $\mathrm{M}=11 \mathrm{~g} / \mathrm{mol}$ )?

We are given both a number $N=1$ and a molecular mass $M=11 \mathrm{~g} / \mathrm{mol}$. Recall that:


$$
m=\frac{N M}{N_{A}}=\frac{(1)(11 \mathrm{~g} / \mathrm{mol})}{6.023 \times 10^{23} \text { atoms } / \mathrm{mol}}
$$

$$
m=1.83 \times 10^{-23} \mathrm{~g}
$$

## Icleal Gas Law

Substituting moles $n$ for mass $m$, we know that:

$$
\frac{P_{1} V_{1}}{n_{1} T_{1}}=\frac{P_{2} V_{2}}{n_{2} T_{2}}
$$

In other words, the ratio $\mathrm{PV} / \mathrm{nT}$ is a constant, and if we can find its value, we can work with a single state.

Since a mole of any gas contains the same number of molecules, it will have the same volume for any gas.

Volume of one mole of a gas:

$$
\mathrm{V}=22.4 \mathrm{~L} \text { or } 22.4 \times 10^{-3} \mathrm{~m}^{3}
$$

## The Universal Gas Constant R

The universal gas constant $R$ is defined as follows:

## $\frac{P V}{n T}=R$

$$
P V=n R T
$$

Evaluate for one mole of gas at 1 atm, $273 \mathrm{~K}, 22.4 \mathrm{~L}$.

$$
R=\frac{P V}{n T}=\frac{(101,300 \mathrm{~Pa})\left(22.4 \times 10^{-3} \mathrm{~m}^{3}\right)}{(1 \mathrm{~mol})(273 \mathrm{~K})}
$$

$$
\mathrm{R}=8.314 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{~K}
$$

Example 5: Two hundred grams of oxygen $(\mathrm{M}=32 \mathrm{~g} / \mathrm{mol})$ fills a 2-L tank at a temperature of $25^{\circ} \mathrm{C}$. What is the absolute pressure P of the gas?

$$
\begin{aligned}
& T=25^{0}+273^{0}=298 K \\
& V=2 L=2 \times 10^{-3} \mathrm{~m}^{3}
\end{aligned}
$$

$$
\mathrm{O}_{2} \begin{aligned}
V & =2 \mathrm{~L} \\
t & =25^{\circ} \mathrm{C} \\
m & =200 \mathrm{~g}
\end{aligned}
$$

$$
P V=n R T
$$


$P=\frac{m R T}{M V}=\frac{(200 \mathrm{~g})(8.314 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{K})(298 \mathrm{~K})}{(32 \mathrm{~g} / \mathrm{mol})\left(2 \times 10^{-3} \mathrm{~m}^{3}\right)}$

$$
\mathrm{P}=7.74 \mathrm{MPa}
$$

Example 6: How many grams of nitrogen gas ( $\mathrm{M}=28 \mathrm{~g} / \mathrm{mol}$ ) will occupy a volume of $2.4 \mathrm{~m}^{3}$ if the absolute pressure is 220 kPa and the temperature is 300 K ?

$V=2.4 \mathrm{~m}^{3}$
$\mathrm{N}_{2} \quad T=300 \mathrm{~K}$
$P=220 \mathrm{kPa}$

$$
\begin{gathered}
m=\frac{P V M}{R T}=\frac{(220,000 \mathrm{~Pa})\left(2.4 \mathrm{~m}^{3}\right)(28 \mathrm{~g} / \mathrm{mol})}{(8.314 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{~K})(300 \mathrm{~K})} \\
m=5930 \mathrm{~g} \text { or } m=5.93 \mathrm{~kg}
\end{gathered}
$$

## Surfmary of Formulas

$$
\frac{P_{1} V_{1}}{m_{1} T_{1}}=\frac{P_{2} V_{2}}{m_{2} T_{2}}
$$

$$
\frac{P_{1} V_{1}}{n_{1} T_{1}}=\frac{P_{2} V_{2}}{n_{2} T_{2}}
$$

$$
n=\frac{N}{N_{A}}
$$

$$
n=\frac{m}{M}
$$

$$
\frac{P V}{n T}=R
$$

$$
P V=n R T
$$



