Topic 5: Electricity and magnetism

5.2 – Heating effect of electric currents

Essential idea: One of the earliest uses for electricity was to produce light and heat. This technology continues to have a major impact on the lives of people around the world.

Nature of science: Although Ohm and Barlow published their findings on the nature of electric current around the same time, little credence was given to Ohm. Barlow's incorrect law was not initially criticized or investigated. This is a reflection of the nature of academia of the time with physics in Germany being largely non-mathematical and Barlow held in high respect in England. It indicates the need for the publication and peer review of research findings in recognized scientific journals.

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Understandings:

- Circuit diagrams
- Kirchhoff's circuit laws
- Heating effect of current and its consequences
- Resistance expressed as $R = V / I$
- Ohm's law
- Resistivity $R = \rho L / A$
- Power dissipation

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Applications and skills:

- Drawing and interpreting circuit diagrams
- Identifying ohmic and non-ohmic conductors through a consideration of the V / I characteristic graph
- Solving problems involving potential difference, current, charge, Kirchhoff's circuit laws, power, resistance and resistivity
- Investigating combinations of resistors in parallel and series circuits
- Describing ideal and non-ideal ammeters and voltmeters
- Describing practical uses of potential divider circuits, including the advantages of a potential divider over a series resistor in controlling a simple circuit

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Applications and skills:

- Investigating one or more of the factors that affect resistivity experimentally

Guidance:

- The filament lamp should be described as a non-ohmic device; a metal wire at a constant temperature is an ohmic device
- The use of non-ideal voltmeters is confined to voltmeters with a constant but finite resistance
- The use of non-ideal ammeters is confined to ammeters with a constant but non-zero resistance
- Application of Kirchhoff's circuit laws will be limited to circuits with a maximum number of two source-carrying loops

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Data booklet reference:

- Kirchhoff's circuit laws:

$$\Sigma V = 0 \text{ (loop)}$$

$$\Sigma I = 0 \text{ (junction)};$$

- $R = V / I$
- $P = VI = I^2R = V^2 / R$
- $R_{\text{total}} = R_1 + R_2 + \ldots$
- $1 / R_{\text{total}} = 1 / R_1 + 1 / R_2 + \ldots$
- $\rho = RA / L$

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International-mindedness:

- A set of universal symbols is needed so that physicists in different cultures can readily communicate ideas in science and engineering

Theory of knowledge:

- Sense perception in early electrical investigations was key to classifying the effect of various power sources, however this is fraught with possible irreversible consequences for the scientists involved. Can we still ethically and safely use sense perception in science research?

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Utilization:

- Although there are nearly limitless ways that we use electrical circuits, heating and lighting are two of the most widespread
- Sensitive devices can employ detectors capable of measuring small variations in potential difference and/or current, requiring carefully planned circuits and high precision components

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Aims:

- **Aim 2:** electrical theory and its approach to macro and micro effects characterizes much of the physical approach taken in the analysis of the universe
- **Aim 3:** electrical techniques, both practical and theoretical, provide a relatively simple opportunity for students to develop a feeling for the arguments of physics
- **Aim 6:** experiments could include (but are not limited to): use of a hot-wire ammeter as an historically important device; comparison of resistivity of a variety of conductors such as a wire at constant temperature, a filament lamp, or a graphite pencil;

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Aims:

- **Aim 6:** determination of thickness of a pencil mark on paper; investigation of ohmic and non-ohmic conductor characteristics; using a resistive wire wound and taped around the reservoir of a thermometer to relate wire resistance to current in the wire and temperature of wire
- **Aim 7:** there are many software and online options for constructing simple and complex circuits quickly to investigate the effect of using different components within a circuit

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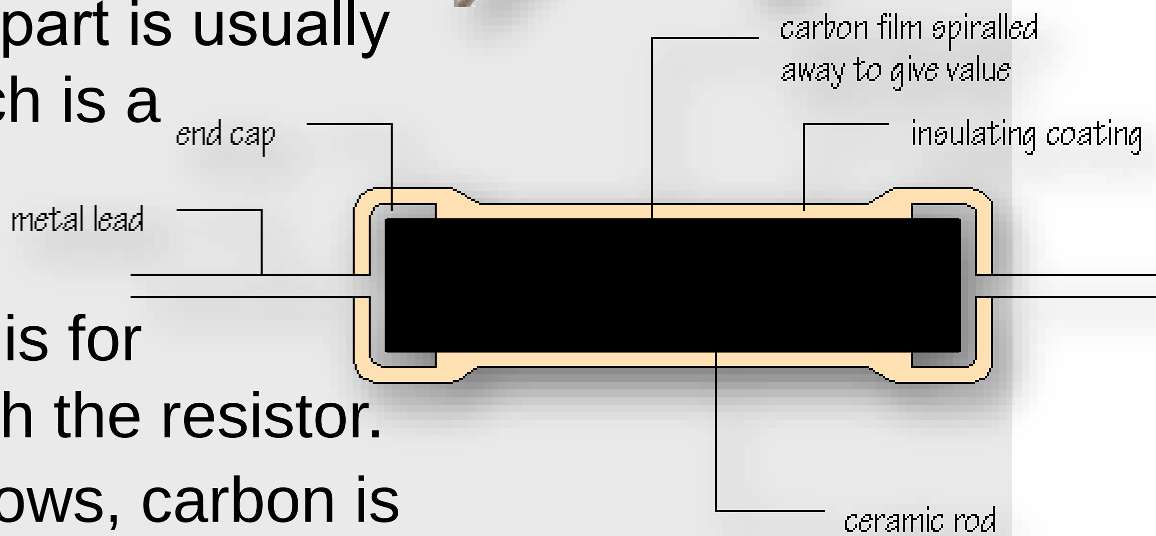
5.2 – Heating effect of electric currents

Resistance

- If you have ever looked inside an electronic device you have no doubt seen what a resistor looks like.
- A resistor's working part is usually made of carbon, which is a semiconductor.



- The less carbon there is, the harder it is for current to flow through the resistor.
- As the animation shows, carbon is spiraled away to cut down the cross-sectional area, thereby increasing the resistance to whatever value is desired.

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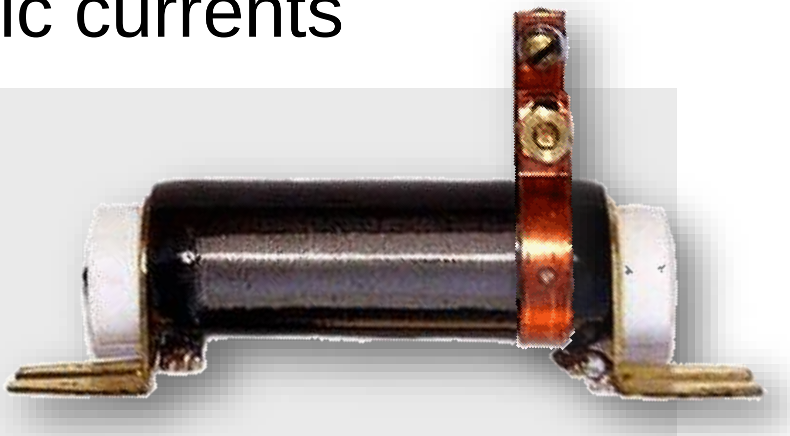
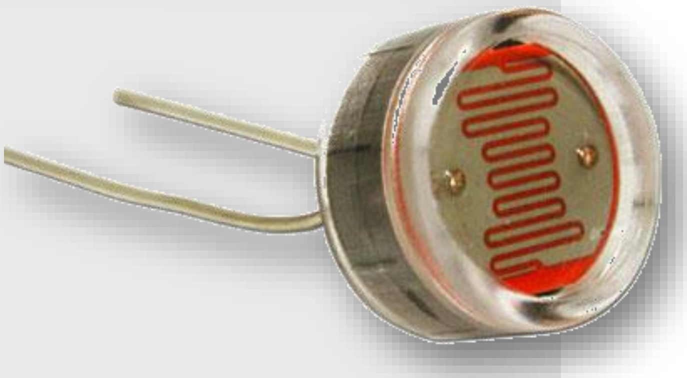
Resistance



- Some very precise resistors are made of wire and are called **wire-wound resistors**.
- And some resistors can be made to vary their resistance by tapping them at various places. These are called **variable resistors** and **potentiometers**.

- **Thermistors** are temperature-dependent resistors, changing their resistance in response to their temperature.



- **Light-dependent resistors (LDRs)** change their resistance in response to light intensity.

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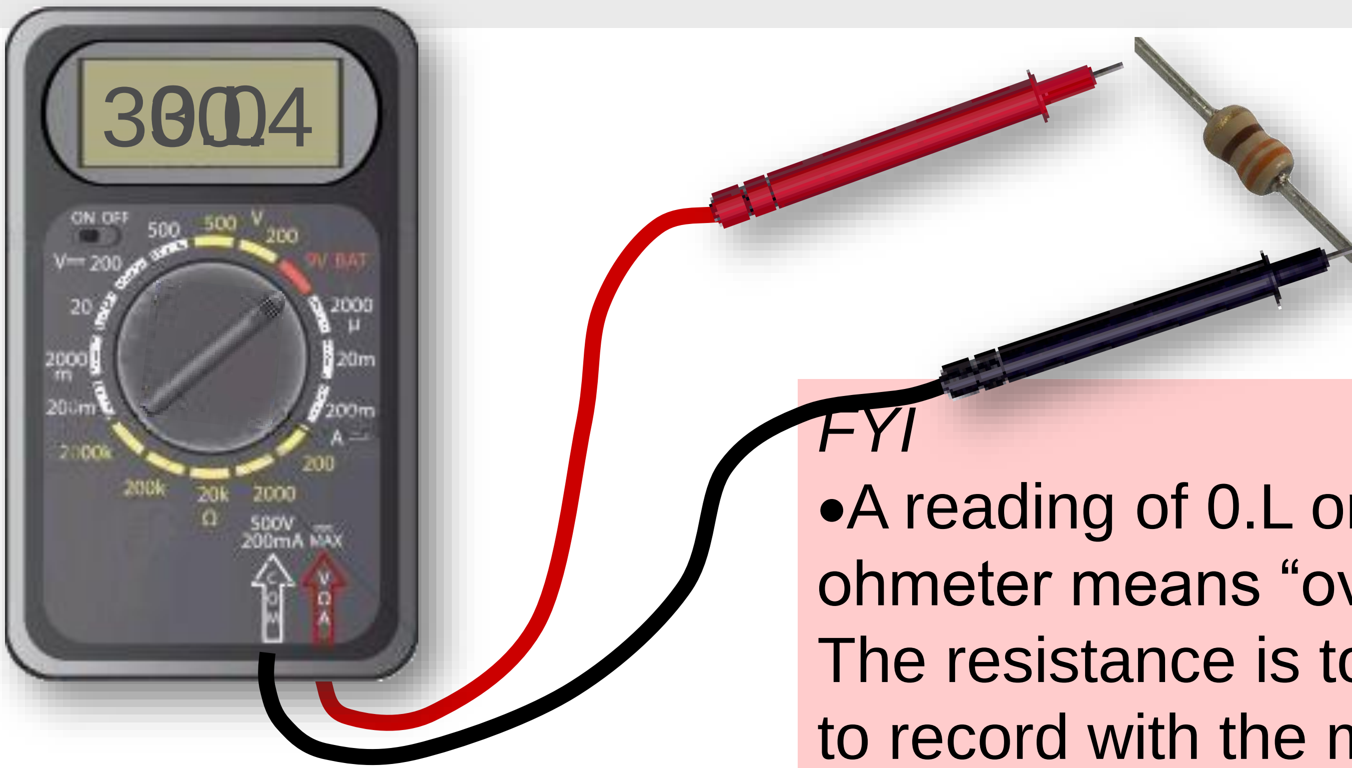
Resistance

- Electrical **resistance** *R* is a measure of how hard it is for current to flow through a material. Resistance is measured in ohms (Ω) using an ohm-meter.



This resistor has a resistance of 330.4 Ω.

FYI

- A reading of 0.L on an ohmeter means “overload”. The resistance is too high to record with the meter.

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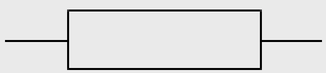
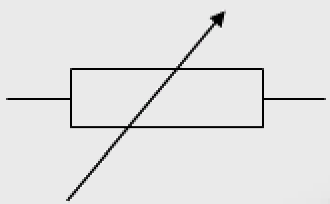
5.2 – Heating effect of electric currents

Resistance

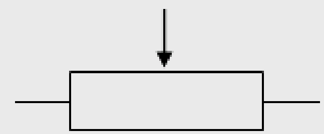
- The different types of resistors have different **schematic symbols**.



fixed-value resistor
2 leads



variable resistor
2 leads



potentiometer
3 leads

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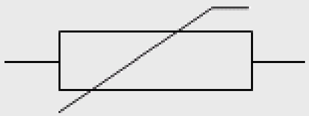
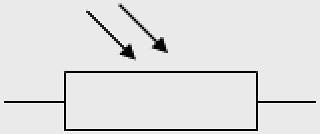
Resistance

- The different types of resistors have different **schematic symbols**.

thermister
2 leads



light-dependent resistor (LDR)
2 leads



As temperature increases resistance decreases

As brightness increases resistance decreases

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Resistor color code	
BLACK	0
BROWN	1
RED	2
ORANGE	3
YELLOW	4
GREEN	5
BLUE	6
VIOLET	7
GRAY	8
WHITE	9

Resistance

- The **resistance** R of a material is the ratio of the potential difference V across the material to the current I flowing through the material.

$R = V / I$	electric resistance

- The units from the formula are ($V\,A^{-1}$) which are called ohms (Ω).

PRACTICE:

A fixed resistor has a current of 18.2 mA when it has a 6.0 V potential difference across it. What is its resistance?

Orange = 3
Orange = 3
Brown = 1

SOLUTION: Last color is **number of zeros**.

- $R = V / I = 6.0 / 18.2 \times 10^{-3} = 330\ \Omega$.

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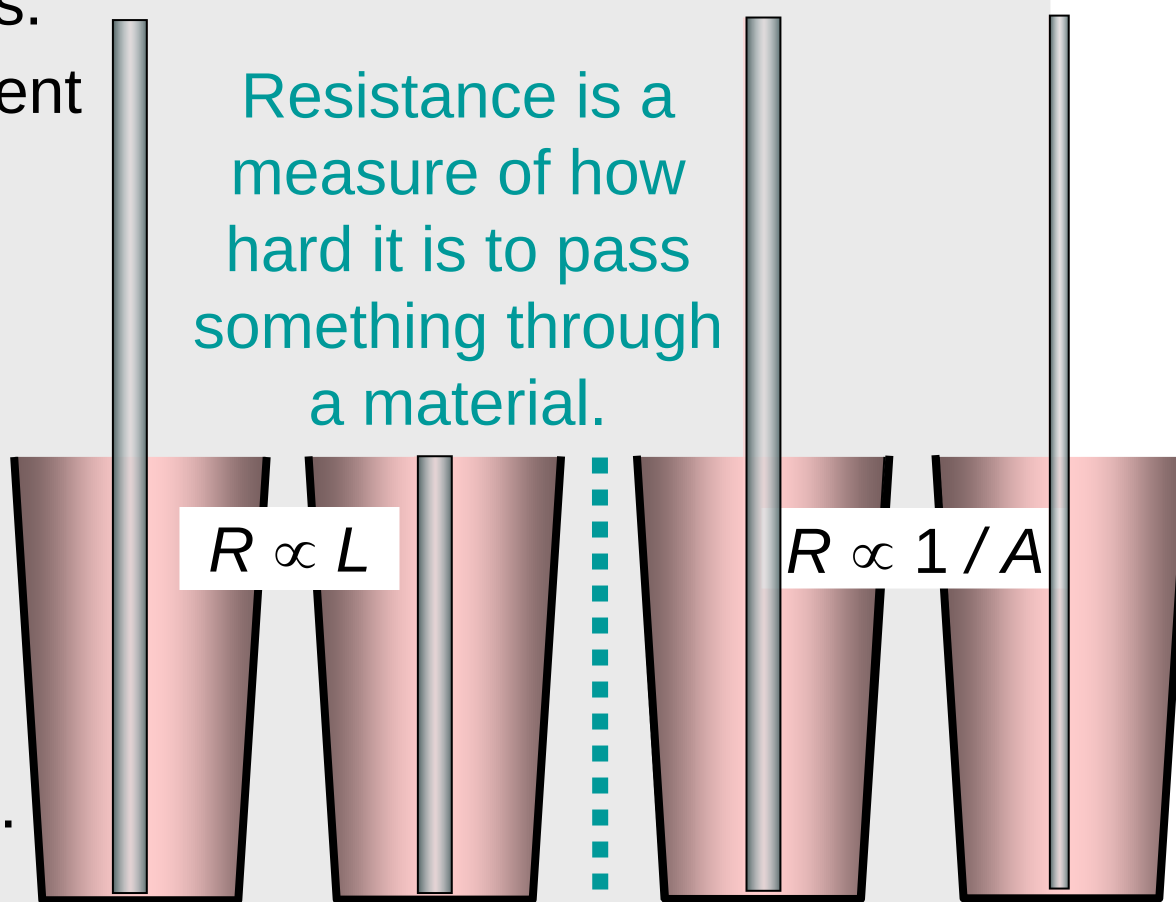
5.2 – Heating effect of electric currents

Resistance

- To understand electrical resistance, consider two identical milk shakes.
- In the first experiment the straws have the same diameter, but different lengths.
- In the second experiment the straws have the same length, but different diameters.
- Note that $R \propto L / A$.

Resistance is a measure of how hard it is to pass something through a material.



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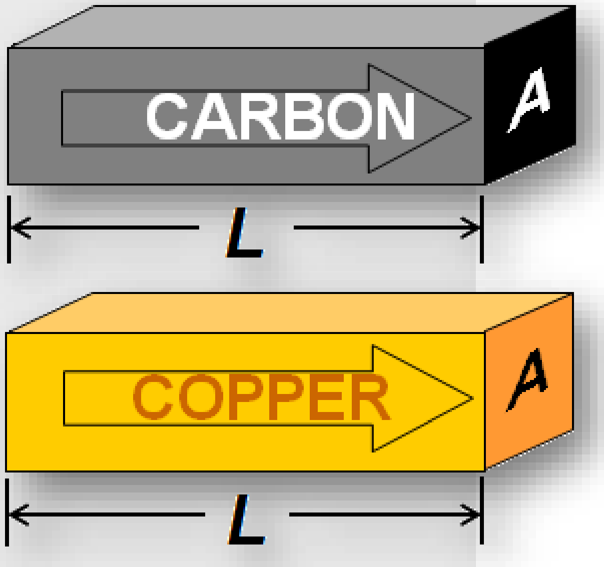
5.2 – Heating effect of electric currents

Resistance

- Of course conductors and resistors are not hollow like straws. And instead of milk shake current we have electrical current.



- Even through solids $R \propto L / A$.
- But R also depends on the material through which the electricity is flowing.
- For example the exact same size of copper will have much less resistance than the carbon.
- With the proportionality constant ρ we have equality:

$R = \rho L / A$ or $\rho = RA / L$	resistance equation

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Resistance

- The Greek ρ is the **resistivity** of the particular material the resistor is made from. It is measured in Ωm.

Resistivities and Temperature Coefficients for Various Materials at 20°C					
Material	ρ ($\Omega \cdot$m)	α (C° $^{-1}$)	Material	ρ ($\Omega \cdot$m)	α (C° $^{-1}$)
Conductors			*Semiconductors*		
Aluminum	2.82×10^{-8}	4.29×10^{-3}	Carbon	3600×10^{-8}	-5.0×10^{-4}
Copper	1.70×10^{-8}	6.80×10^{-3}	Germanium	4.6×10^{-1}	-5.0×10^{-2}
Iron	10×10^{-8}	6.51×10^{-3}	Silicon	2.5×10^{2}	-7.0×10^{-2}
Mercury	98.4×10^{-8}	0.89×10^{-3}			
Nichrome	100×10^{-8}	0.40×10^{-3}	*Nonconductors*		
Nickel	7.8×10^{-8}	6.0×10^{-3}	Glass	10^{12}	
Platinum	10×10^{-8}	3.93×10^{-3}	Rubber	10^{15}	
Silver	1.59×10^{-8}	6.1×10^{-3}	Wood	10^{10}	
Tungsten	5.6×10^{-8}	4.5×10^{-3}			

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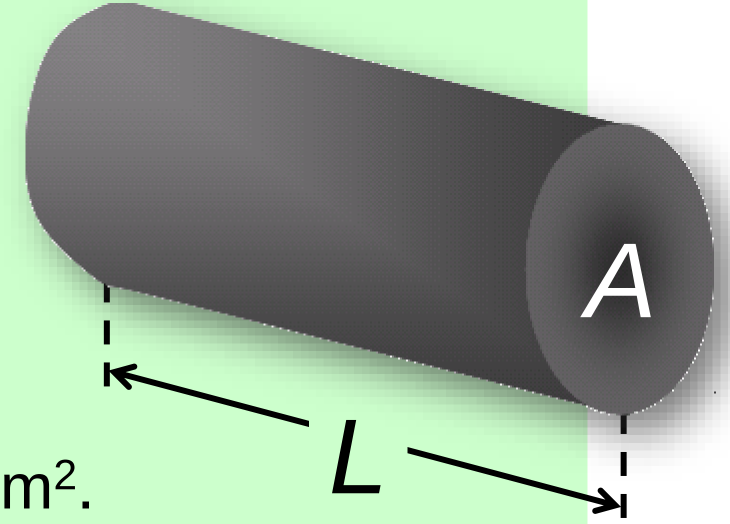
Resistance

- Note that resistance depends on temperature. The IBO does not require us to explore this facet of resistivity.

PRACTICE: What is the resistance of a 0.00200 meter long carbon core resistor having a core diameter of 0.000100 m? Assume the temperature is 20 °C.



- $r = d / 2 = 0.0001 / 2 = 0.00005$ m.
- $A = \pi r^2 = \pi(0.00005)^2 = 7.854\times10^{-9}$ m^2.
- From the table $\rho = 3600\times10^{-8}\ \Omega\,\mathrm{m}$.

$$R = \rho L / A$$

$$= (3600\times10^{-8})(0.002) / 7.854\times10^{-9} = 9.17\ \Omega.$$

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Ohm's law

- The German Ohm studied resistance of materials in the 1800s and in 1826 stated:

"Provided the temperature is kept constant, the resistance of very many materials is constant over a wide range of applied potential differences, and therefore the potential difference is proportional to the current."

- In formula form Ohm's law looks like this:

$V \propto I$ or $V / I = \text{CONST}$ or $V = IR$	Ohm's law

FYI

- Ohm's law applies to components with constant R.

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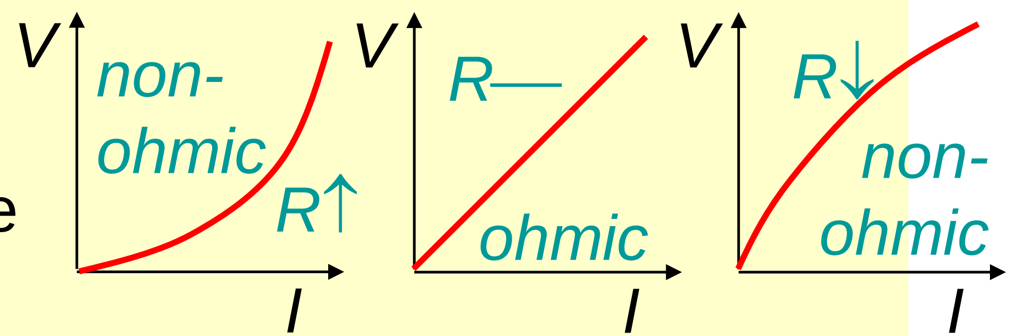
Ohm's law – ohmic and non-ohmic behavior

- A material is considered **ohmic** if it behaves according to Ohm's law. In other words the resistance stays constant as the voltage changes.

EXAMPLE: Label appropriate V-I graphs with the following labels: ohmic, non-ohmic, *R* increasing, *R* decreasing, *R* constant.



SOLUTION:

- First label the resistance dependence.
- $R = V / I$ so R is just the slope of the V vs. I graph.
- Ohm's law states the R is constant. Thus only one graph is ohmic.

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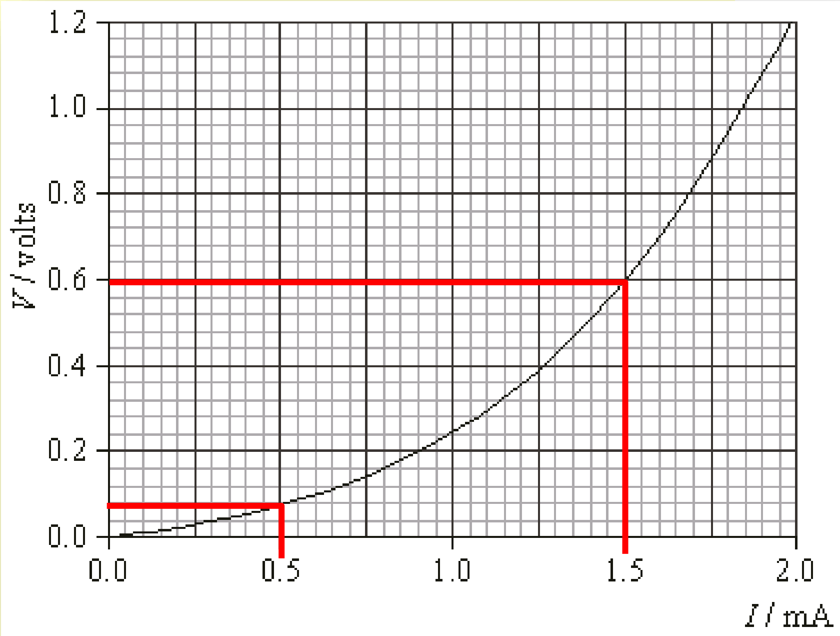
Ohm's law – ohmic and non-ohmic behavior

EXAMPLE: The graph shows the applied voltage V vs. resulting current I through a tungsten filament lamp.



Find R when I = 0.5 mA and 1.5 mA. Is this filament ohmic or non-ohmic?

SOLUTION:

- At 0.5 mA: V = 0.08 V

  $R = V / I = 0.08 / 0.5\times10^{-3} = 160\ \Omega$.

- At 1.5 mA: V = 0.6 V

  $R = V / I = 0.6 / 1.5\times10^{-3} = 400\ \Omega$.

Since R is not constant the filament is non-ohmic.

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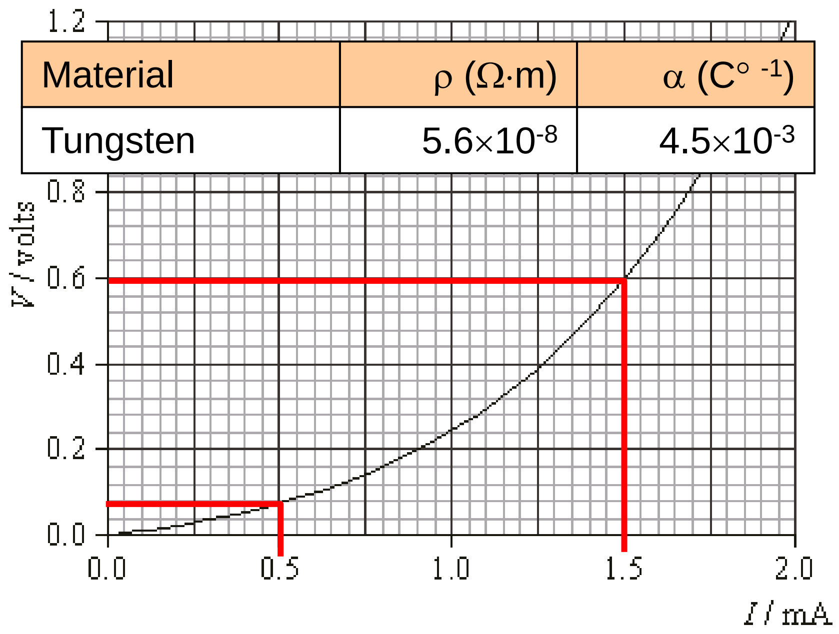
Ohm's law – ohmic and non-ohmic behavior

EXAMPLE: The graph shows the applied voltage *V* vs. resulting current *I* through a tungsten filament lamp.



Material	ρ (Ω·m)	α (C° $^{-1}$)
Tungsten	5.6×10^{-8}	4.5×10^{-3}

Explain why a lamp filament might be non-ohmic.

SOLUTION:

- The temperature coefficient for tungsten is positive, typical for conductors.
- Therefore, the hotter the filament the higher *R*.
- But the more current, the hotter a lamp filament burns.
- Thus, the bigger the *I* the bigger the *R*.

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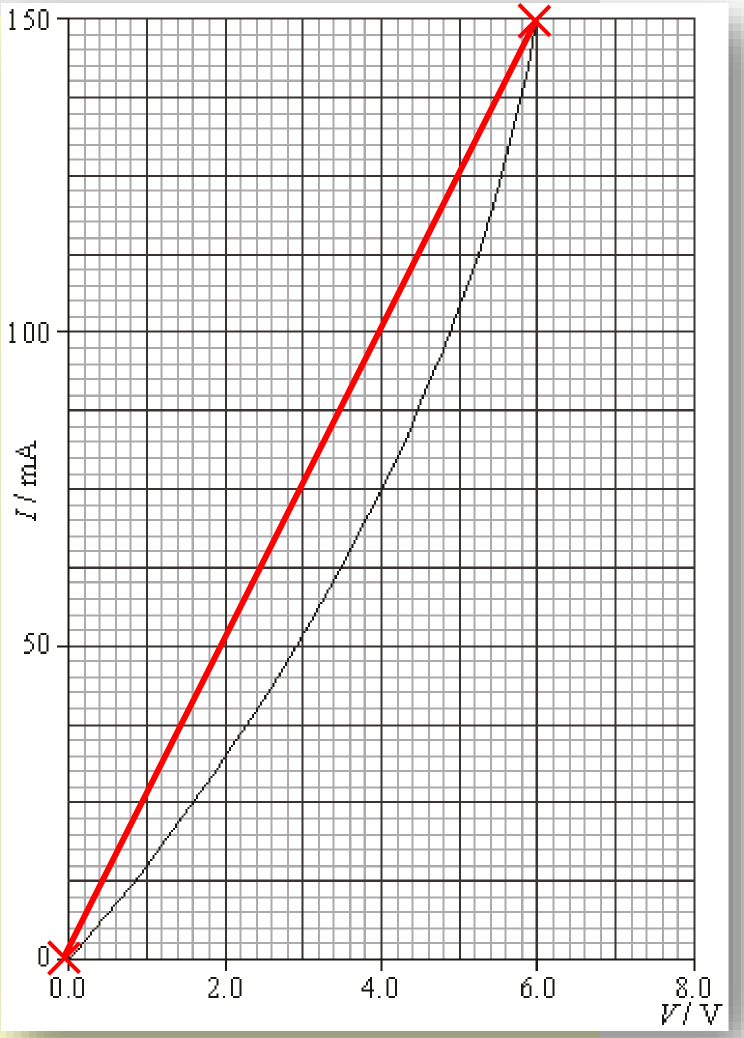
Ohm's law – ohmic and non-ohmic behavior

EXAMPLE: The *I*-*V* characteristic is shown for a non-ohmic component. Sketch in the *I*-*V* characteristic for a 40 Ω ohmic component in the range of 0.0 V to 6.0 V.



SOLUTION:

- "Ohmic" means $V = IR$ and R is constant (and the graph is linear).
- Thus $V = I \times 40$ or $I = V / 40$.
- If $V = 0$, $I = 0 / 40 = 0.0$.
- If $V = 6$, $I = 6 / 40 = 0.15$ A.
- But 0.15 A = 150 mA.

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Power dissipation

- Recall that **power** is the rate at which work is being done. Thus $P = W / t$.
- From Topic 5.1 we learned that $W = qV$.
- Thus

$$P = W / t$$

$$P = qV / t$$

$$P = (q / t)V$$

$$P = IV.$$

FYI

- This power represents the energy per unit time delivered to, or consumed by, an electrical component having a current I and a potential difference V.

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Power dissipation

PRACTICE: Use the definition of resistance $R = V / I$. together with the one we just derived ($P = VI$) to derive the following two formulas:

(a) $P = I^2R$

(b) $P = V^2/ R$.

SOLUTION:

(a) From $R = V / I$ we get $V = IR$.

$$P = IV = I (IR) = I^2R.$$

(b) From $R = V / I$ we get $I = V / R$.

$$P = IV = (V / R) (V) = V^2/ R.$$

$P = VI = I^2R = V^2/ R$	electrical power

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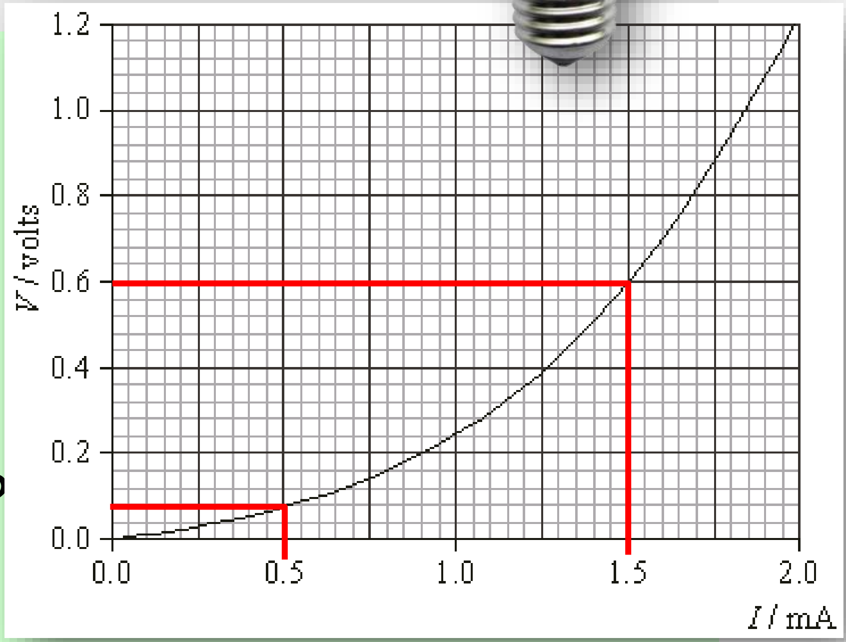
Power dissipation

PRACTICE:

The graph shows the *V*-*I* characteristics of a tungsten filament lamp.

What is its power consumption at I = 0.5 mA and at I = 1.5 mA?



SOLUTION:

- At 0.5 mA, V = 0.08 V.
- $P = IV = (0.5\times10^{-3})(0.08) = 4.0\times10^{-5}$ W.
- At 1.5 mA, V = 0.6 V.
- $P = IV = (1.5\times10^{-3})(0.6) = 9.0\times10^{-4}$ W.

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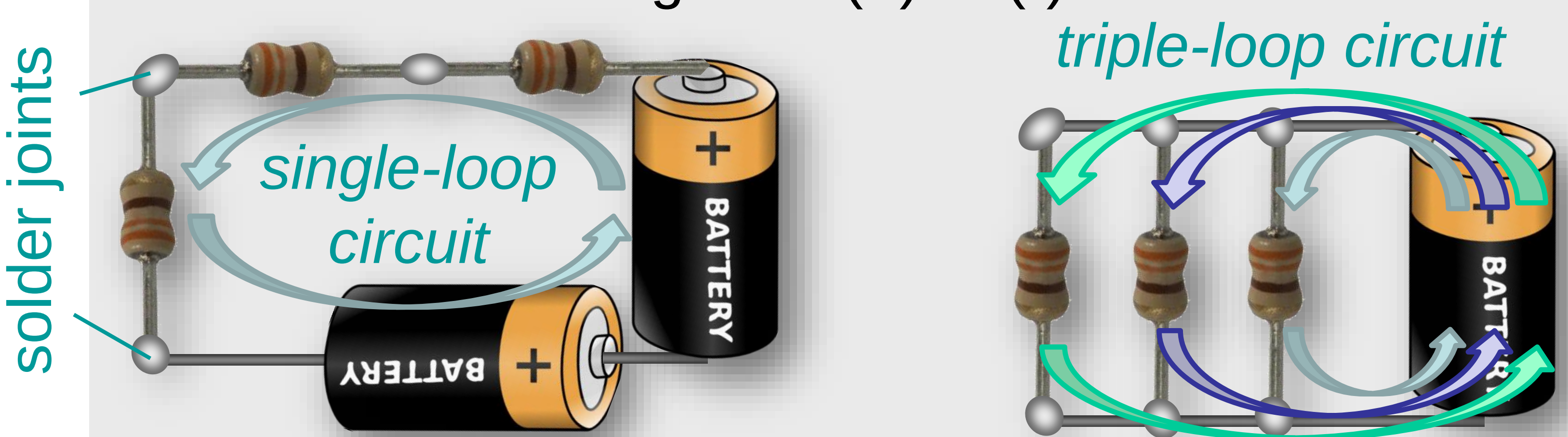
5.2 – Heating effect of electric currents

Electric circuits

- An **electric circuit** is a set of conductors (like wires) and components (like resistors, lights, etc.) connected to an electrical voltage source (like a cell or a battery) in such a way that current can flow in complete loops.
- Here are two circuits consisting of cells, resistors, and wires.
- Note current flowing from (+) to (-) in each circuit.



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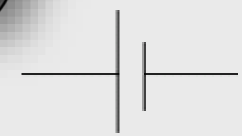
Circuit diagrams

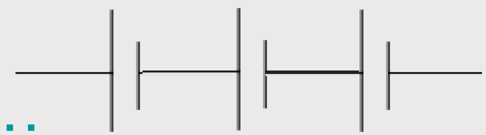
•A complete circuit will always contain a cell or a battery.

this is really a cell…

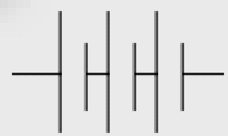
•The **schematic diagram** of a cell is this:



this is a battery…



•A **battery** is just *a group of cells connected in series*:

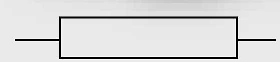


this is the same battery…

•If each cell is 1.5 V, then the battery above is 3(1.5) = 4.5 V. What is the voltage of your calculator battery?

•A fixed-value resistor looks like this:

•The schematic of a fixed-value resistor looks like this:



this is a resistor…

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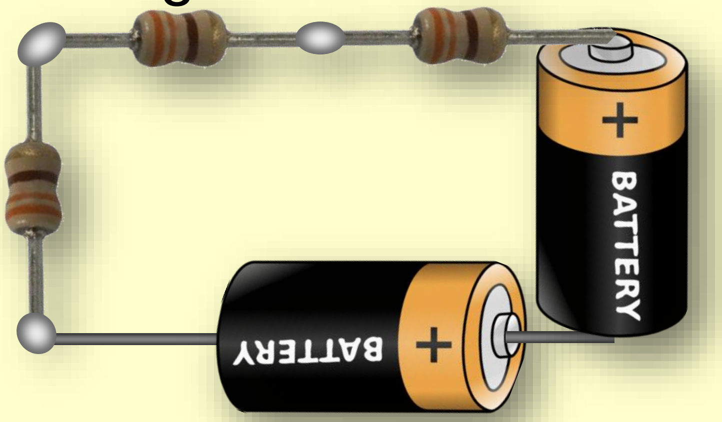
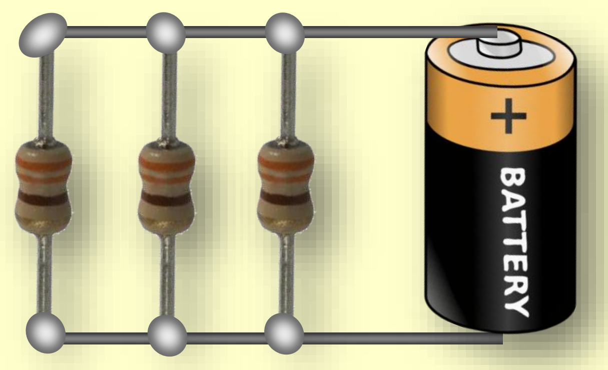
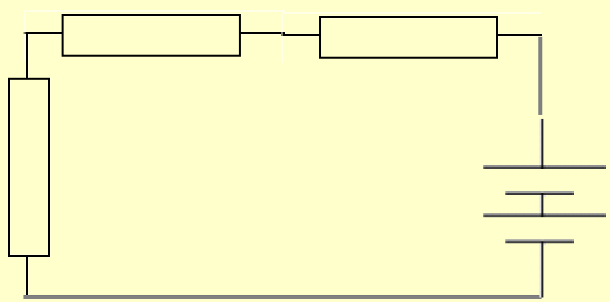
Drawing and interpreting circuit diagrams

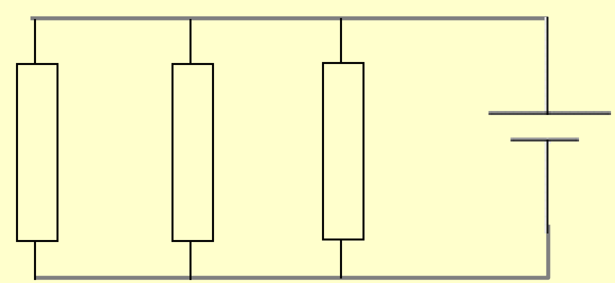
EXAMPLE: Draw schematic diagrams of each of the following circuits:





SOLUTION:





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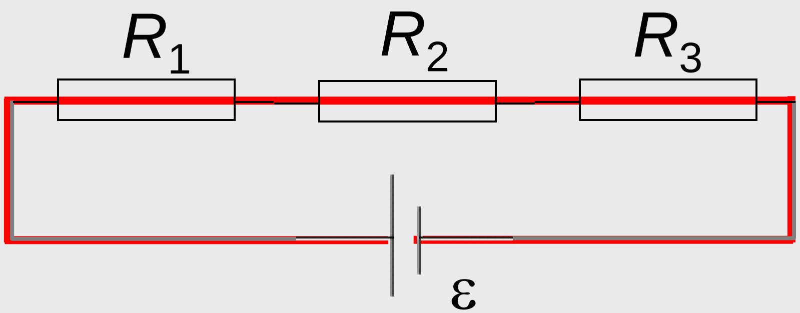
Investigating combinations of resistors in series

- Resistors can be connected to one another in series, which means one after the other.



- Note that there is only one current I and that I is the same for all series components.
- Conservation of energy tells us $\cancel{q}\varepsilon = \cancel{q}V_1 + \cancel{q}V_2 + \cancel{q}V_3$.
- Thus $\varepsilon = IR_1 + IR_2 + IR_3$ from Ohm's law $V = IR$

$\varepsilon = I(R_1 + R_2 + R_3)$ factoring out I

$\varepsilon = I(R)$, where $R = R_1 + R_2 + R_3$.

$R = R_1 + R_2 + \ldots$	equivalent resistance in series

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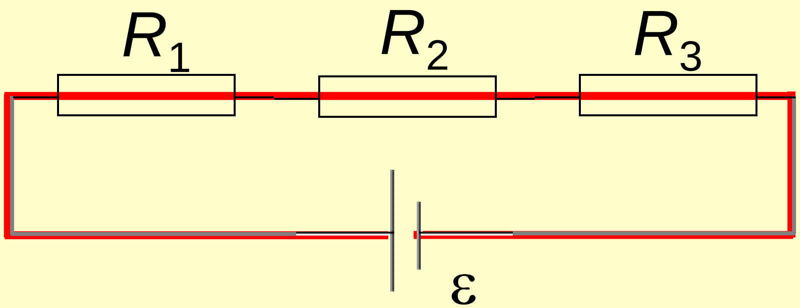
Investigating combinations of resistors in series

EXAMPLE: Three resistors of 330 Ω each are connected to a 6.0 V battery in series as shown.



(a) What is the circuit's equivalent resistance?

(b) What is the current in the circuit?

SOLUTION:

(a) In series, $R = R_1 + R_2 + R_3$ so that

$$R = 330 + 330 + 330 = 990\ \Omega.$$

(b) Since the voltage on the entire circuit is 6.0 V, and since the total resistance is 990 Ω, from Ohm's law we have $I = V / R = 6 / 990 =$ 0.0061 A.

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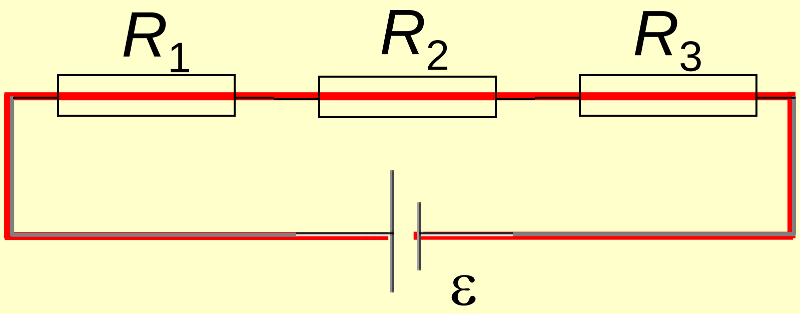
Investigating combinations of resistors in series

EXAMPLE: Three resistors of 330 Ω each are connected to a 6.0 V battery in series as shown.



(c) What is the voltage on each resistor?

SOLUTION:

(c) The current I we just found is the same everywhere. Thus each resistor has a current of $I = 0.0061$ A.

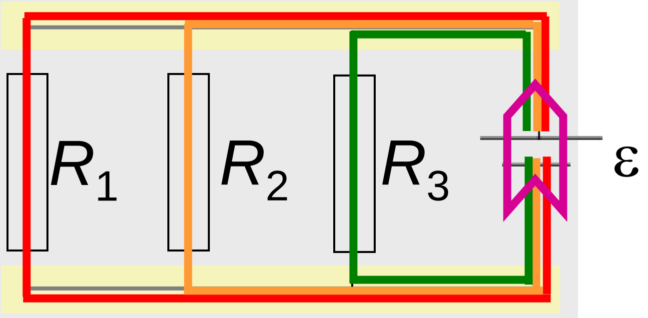
•From Ohm's law, each resistor has a voltage given by $V = IR = (0.0061)(330) = 2.0$ V.

FYI

•In series the V's are different if the R's are different.

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

Investigating combinations of resistors in parallel



- Resistors can also be in parallel.
- In this circuit each resistor is connected directly to the cell.
- Thus each resistor has the same voltage V and V is the same for all parallel components.
- We can then write $\varepsilon = V_1 = V_2 = V_3 \equiv V$.
- But there are three currents I_1, I_2, and I_3.
- Since the total current I passes through the cell we see that $I = I_1 + I_2 + I_3$.
- If R is the equivalent or total resistance of the three resistors, then $I = I_1 + I_2 + I_3$ becomes

$\varepsilon / R = V_1 / R_1 + V_2 / R_2 + V_3 / R_3$ Ohm's law $I = V / R$

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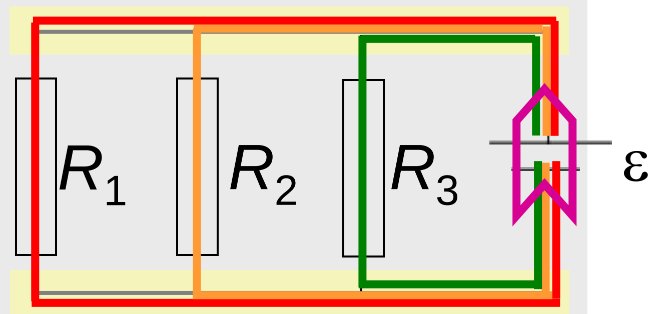
5.2 – Heating effect of electric currents

Investigating combinations of resistors in parallel



- Resistors can also be in parallel.
- In this circuit each resistor is connected directly to the cell.
- Thus each resistor has the same voltage V and V is the same for all parallel components.
- From $\varepsilon = V_1 = V_2 = V_3 \equiv V$

and $\varepsilon / R = V_1 / R_1 + V_2 / R_2 + V_3 / R_3$,

we get $\cancel{V} / R = \cancel{V} / R_1 + \cancel{V} / R_2 + \cancel{V} / R_3$.

- Thus the equivalent resistance R is given by

$1 / R = 1 / R_1 + 1 / R_2 + \ldots$	equivalent resistance in parallel

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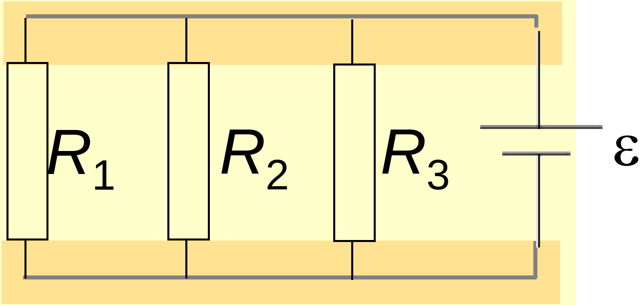
5.2 – Heating effect of electric currents

Investigating combinations of resistors in parallel

EXAMPLE: Three resistors of 330 Ω each are connected to a 6.0 V cell in parallel as shown.



(a) What is the circuit's resistance?

(b) What is the voltage on each resistor?

SOLUTION:

(a) In parallel, $1 / R = 1 / R_1 + 1 / R_2 + 1 / R_3$ so that

$$1 / R = 1 / 330 + 1 / 330 + 1 / 330 = 0.00909.$$

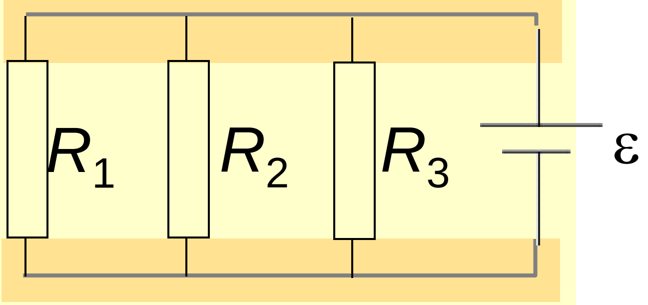
Thus $R = 1 / 0.00909 = 110\ \Omega$.

(b) The voltage on each resistor is 6.0 V, since the resistors are in parallel. (Each resistor is clearly directly connected to the battery).

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

Investigating combinations of resistors in parallel

EXAMPLE: Three resistors of 330 Ω each are connected to a 6.0 V cell in parallel as shown.



(c) What is the current in each resistor?

SOLUTION:

(c) Using Ohm's law ($I = V / R$):

$$I_1 = V_1 / R_1 = 6 / 330 = 0.018 \text{ A.}$$
$$I_2 = V_2 / R_2 = 6 / 330 = 0.018 \text{ A.}$$
$$I_3 = V_3 / R_3 = 6 / 330 = 0.018 \text{ A.}$$

FYI

- In parallel the I's are different if the R's are different.

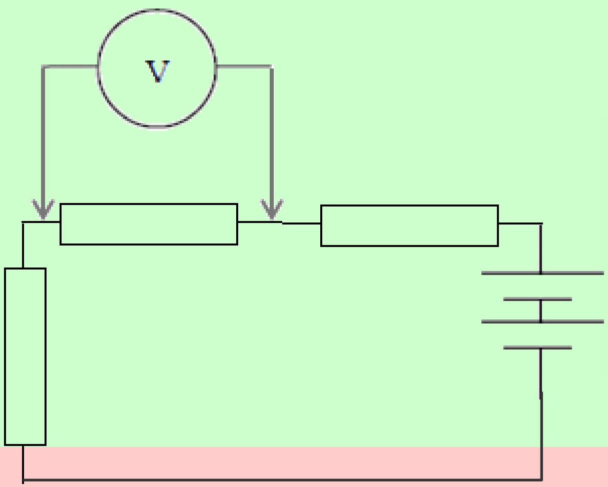
Topic 5: Electricity and magnetism

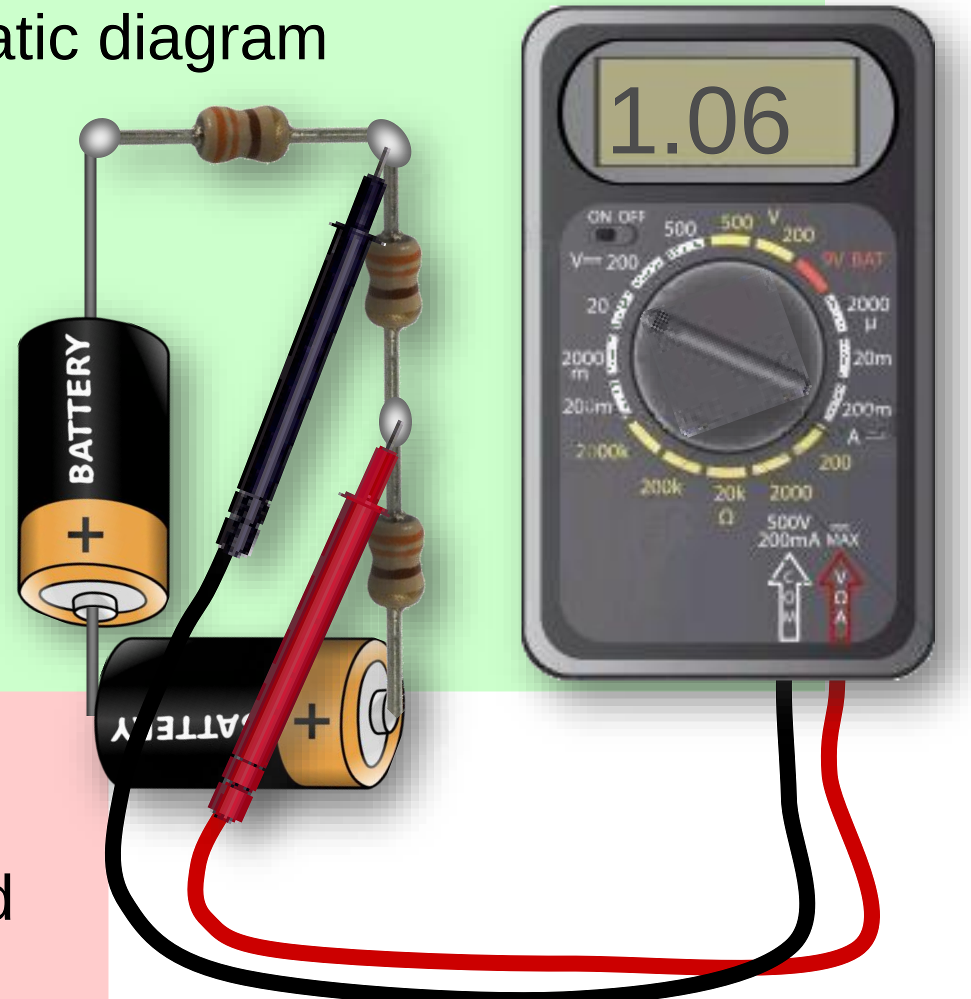
5.2 – Heating effect of electric currents

Circuit diagrams - voltmeters are connected in parallel

PRACTICE: Draw a schematic diagram for this circuit:

SOLUTION:





FYI

- Be sure to position the voltmeter across the desired resistor **in parallel**.

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

Circuit diagrams - voltmeters are connected in parallel

EXAMPLE:

A battery's voltage is measured as shown.

(a) What is the uncertainty in it's measurement?

SOLUTION:

- For digital devices always use the place value of the least significant digit as your raw uncertainty.
- For this voltmeter the voltage is measured to the tenths place so we give the raw uncertainty a value of $\Delta V = \pm 0.1$ V.

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

Circuit diagrams - voltmeters are connected in parallel

EXAMPLE:

A battery's voltage is measured as shown.

(b) What is the fractional error in this measurement?

SOLUTION: Fractional error is just $\Delta V / V$. For this particular measurement we then have

- $\Delta V / V = 0.1 / 9.4 = 0.011$ (or 1.1%).

FYI

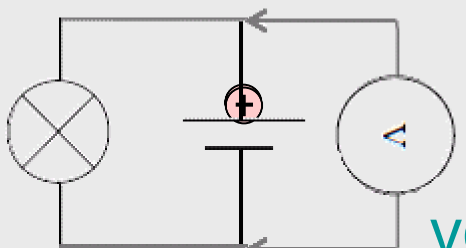
- When using a voltmeter the red lead is placed at the point of highest potential.

Topic 5: Electricity and magnetism

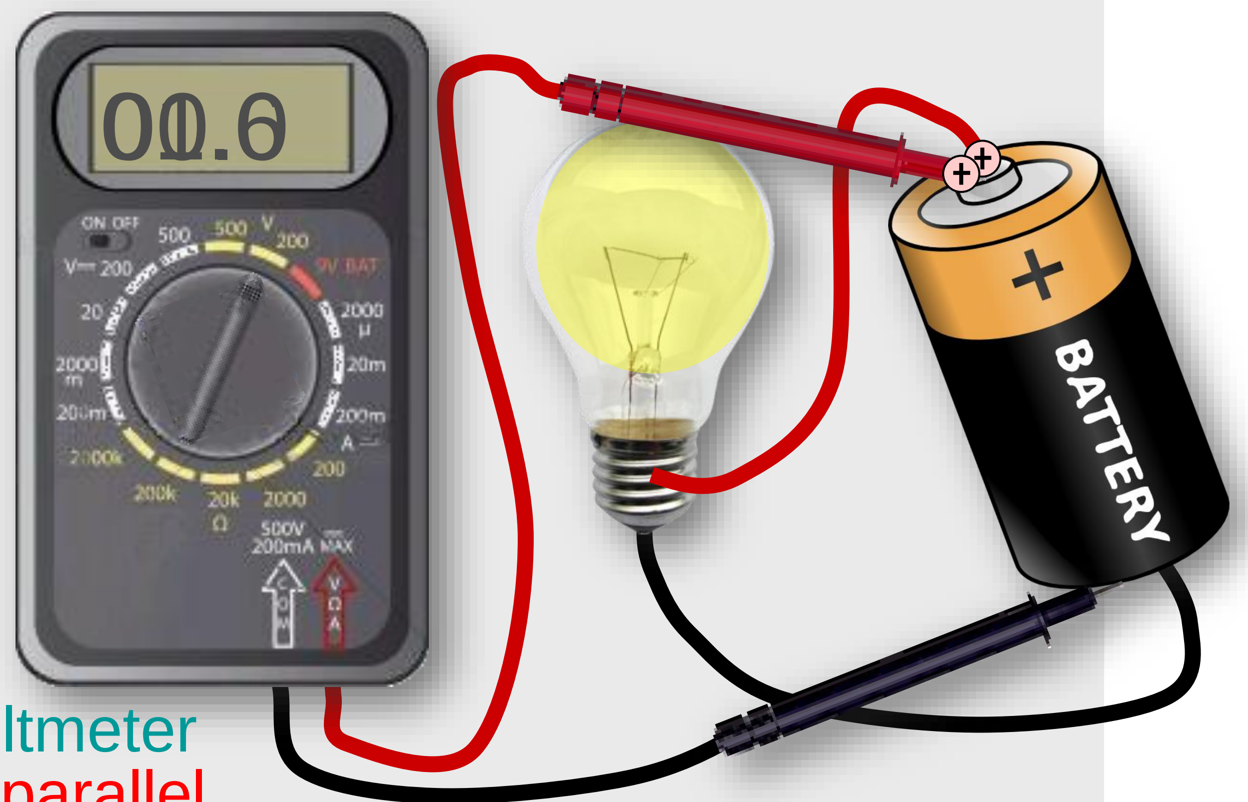
5.2 – Heating effect of electric currents

Circuit diagrams - voltmeters are connected in parallel

- Consider the simple circuit of battery, lamp, and wire.
- To measure the **voltage** of the circuit we merely connect the voltmeter while the circuit is in operation.



lamp cell



voltmeter
in parallel

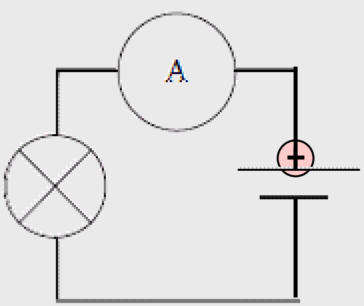
Topic 5: Electricity and magnetism

5.2 – Heating effect of electric currents

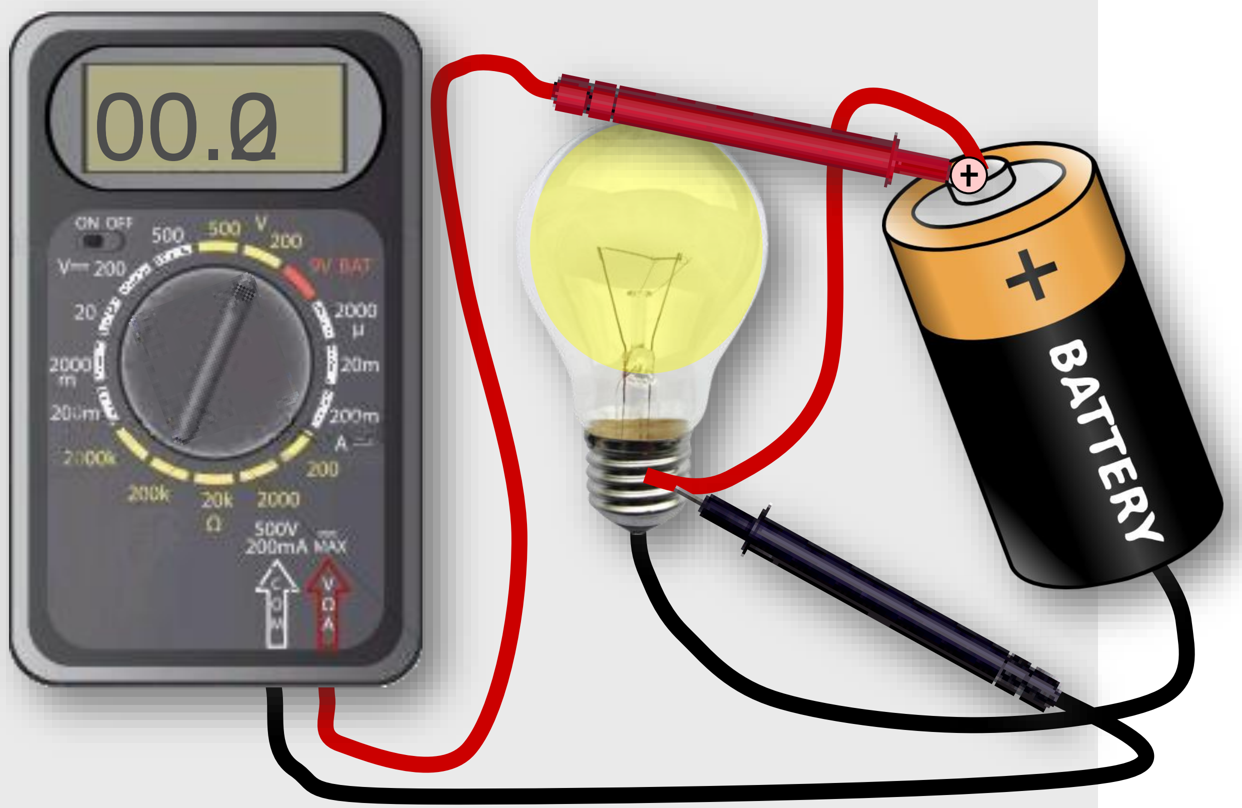
Circuit diagrams - ammeters are connected in series

- To measure the **current** of the circuit we must break the circuit and insert the ammeter so that it intercepts all of the electrons that normally travel through the circuit.

ammeter
in series



lamp cell



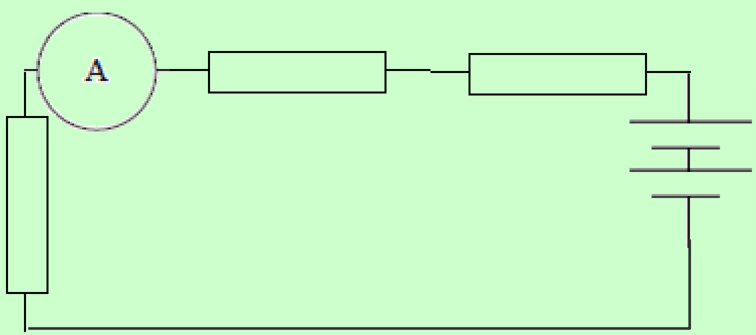
Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

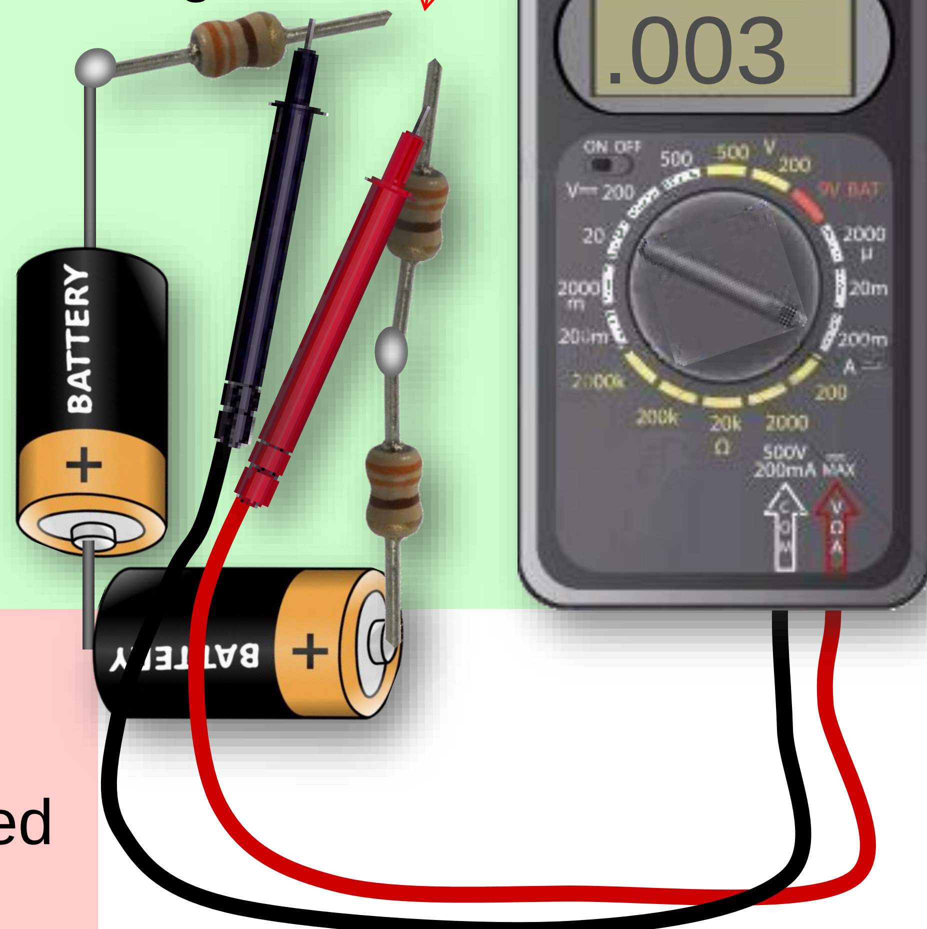
the circuit must be temporarily broken to insert the ammeter

Circuit diagrams - ammeters are connected in series

PRACTICE: Draw a schematic diagram for this circuit:

SOLUTION:





FYI

- Be sure to position the ammeter between the desired resistors **in series**.

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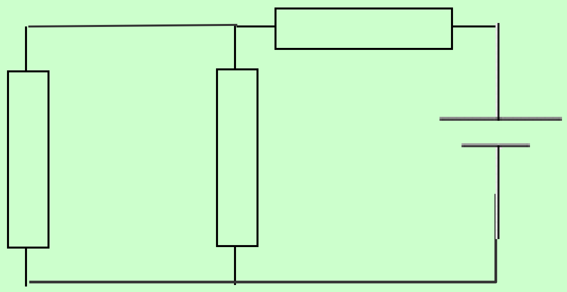
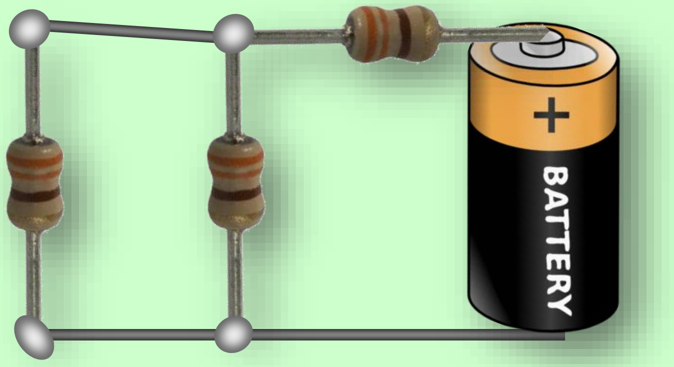
5.2 – Heating effect of electric currents

Circuit diagrams

PRACTICE: Draw a schematic diagram for this circuit:

SOLUTION:



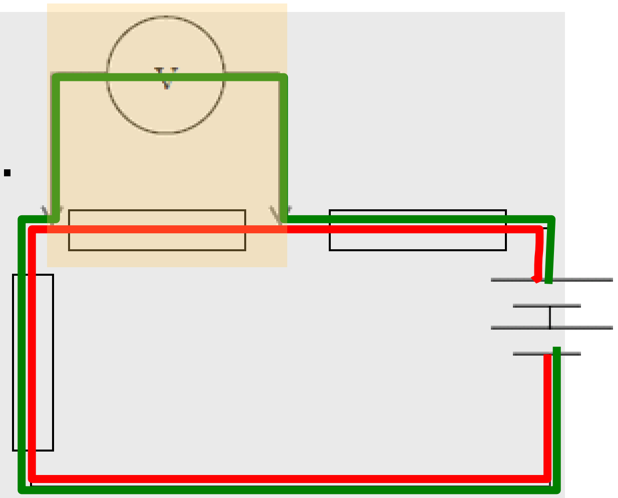


FYI

- This circuit is a combination series-parallel. In a later slide you will learn how to find the equivalent resistance of the combo circuit.

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

Ideal voltmeters - ∞ Ω resistance



- Voltmeters are connected in parallel.
- The voltmeter reads the voltage of only the component it is in parallel with.
- The green current represents the amount of current the battery needs to supply to the voltmeter in order to make it register.
- The red current is the amount of current the battery supplies to the *original circuit*.
- In order to NOT ALTER the original properties of the circuit, ideal voltmeters have extremely high resistance (∞ Ω) to minimize the green current.

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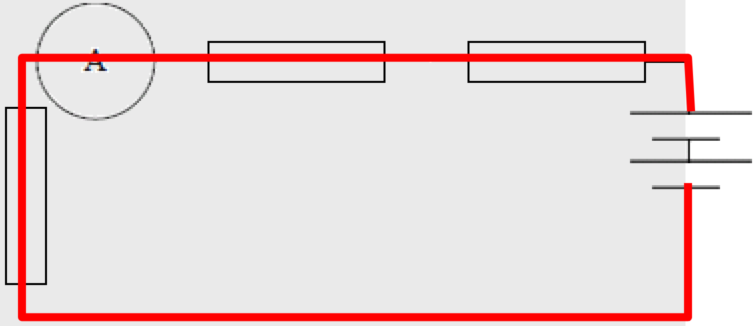
Ideal ammeters - 0 Ω resistance



- Ammeters are connected in series.
- The ammeter is supposed to read the current of the original circuit.
- In order to NOT ALTER the original properties of the circuit, ideal ammeters have extremely low resistance (0 Ω) to minimize the effect on the red current.

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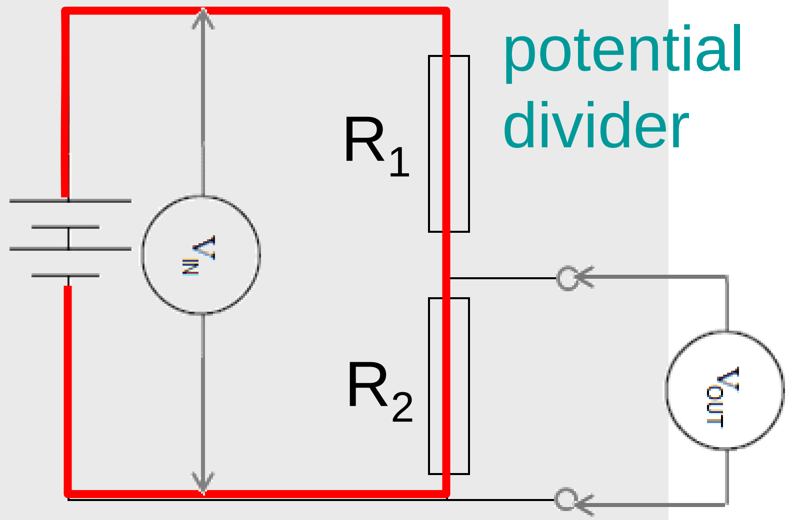
5.2 – Heating effect of electric currents

Potential divider circuits



- Consider a battery of $\varepsilon = 6$ V. Suppose we have a light bulb that can only use three volts. How do we obtain 3 V from a 6 V battery?
- A **potential divider** is a circuit made of two (or more) series resistors that allows us to tap off any voltage we want that is less than the battery voltage.
- The **input voltage** is the emf of the battery.
- The **output voltage** is the voltage drop across R_2.
- Since the resistors are in series $R = R_1 + R_2$.

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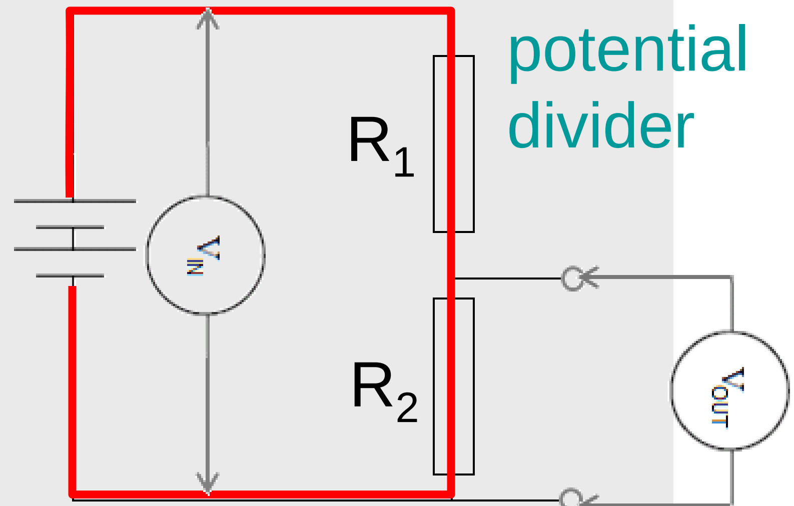
5.2 – Heating effect of electric currents

Potential divider circuits



- Consider a battery of $\varepsilon = 6$ V. Suppose we have a light bulb that can only use three volts. How do we obtain 3 V from a 6 V battery?
- From Ohm's law the current I of the divider is given by $I = V_{IN} / R = V_{IN} / (R_1 + R_2)$.
- But $V_{OUT} = V_2 = IR_2$ so that

$$V_{OUT} = IR_2$$

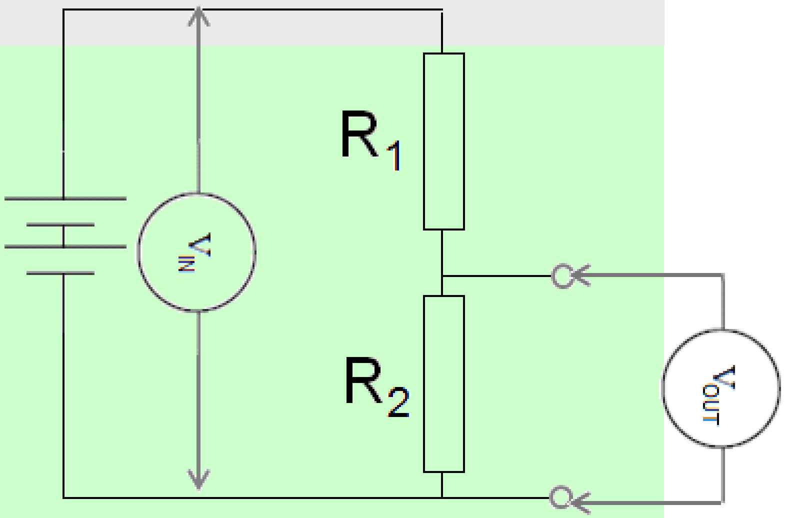
$$= R_2 \times V_{IN} / (R_1 + R_2).$$

$$V_{OUT} = V_{IN} [\, R_2 / (R_1 + R_2) \,]$$ **potential divider**

Topic 5: Electricity and magnetism

5.2 – Heating effect of electric currents

Potential divider circuits



PRACTICE:

Find the output voltage if the battery has an emf of 9.0 V, R_1 is a 2200 Ω resistor, and R_2 is a 330 Ω resistor.

SOLUTION:

- Use $V_{OUT} = V_{IN} [R_2 / (R_1 + R_2)]$

$V_{OUT} = 9 [330 / (2200 + 330)]$

$V_{OUT} = 9 [330 / 2530] = 1.2$ V.

FYI

- The bigger R_2 is in comparison to R_1, the closer V_{OUT} is in proportion to the total voltage.

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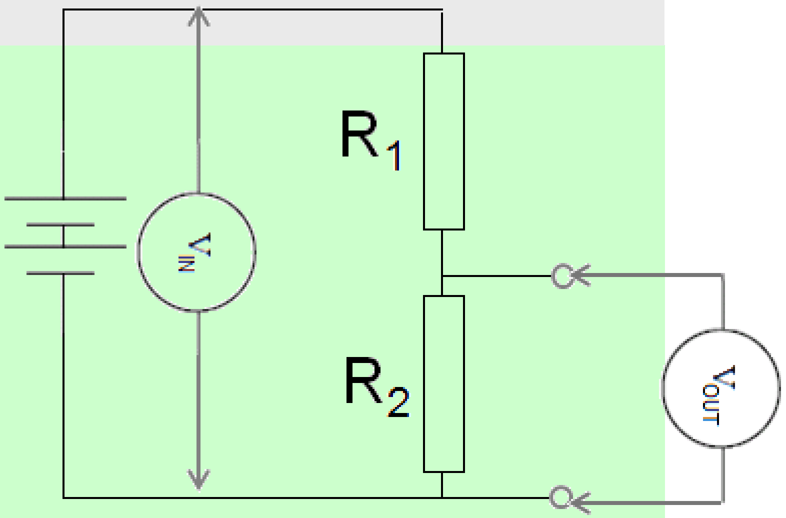
Potential divider circuits



PRACTICE:

Find the value of R_2 if the battery has an emf of 9.0 V, R_1 is a 2200 Ω resistor, and we want an output voltage of 6 V.

SOLUTION:

- Use the formula $V_{OUT} = V_{IN}\,[\,R_2 / (R_1 + R_2)\,]$. Thus

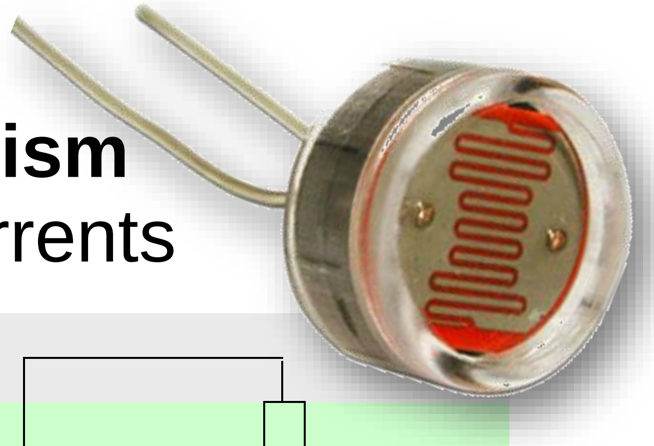
$$6 = 9\,[\,R_2 / (2200 + R_2)\,]$$

$$6(2200 + R_2) = 9R_2$$

$$13200 + 6R_2 = 9R_2$$

$$13200 = 3R_2$$

$$R_2 = 4400\ \Omega$$



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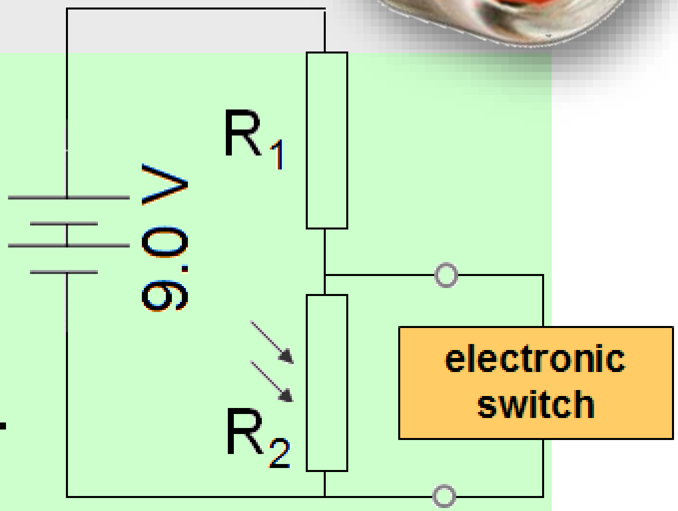
5.2 – Heating effect of electric currents

Potential divider circuits



PRACTICE: A light-dependent resistor (LDR) has R = 25 Ω in bright light and R = 22000 Ω in low light. An electronic switch will turn on a light when its p.d. is above 7.0 V. What should the value of R_1 be?

SOLUTION: Use $V_{OUT} = V_{IN} [\, R_2 / (R_1 + R_2) \,]$. Thus

$$7 = 9\,[\,22000 / (R_1 + 22000)\,]$$

$$7(R_1 + 22000) = 9(22000)$$

$$7R_1 + 154000 = 198000$$

$$7R_1 = 44000$$

$$R_1 = 6300\ \Omega \qquad (6286)$$

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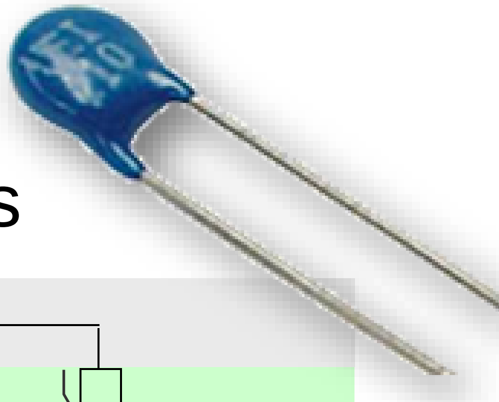
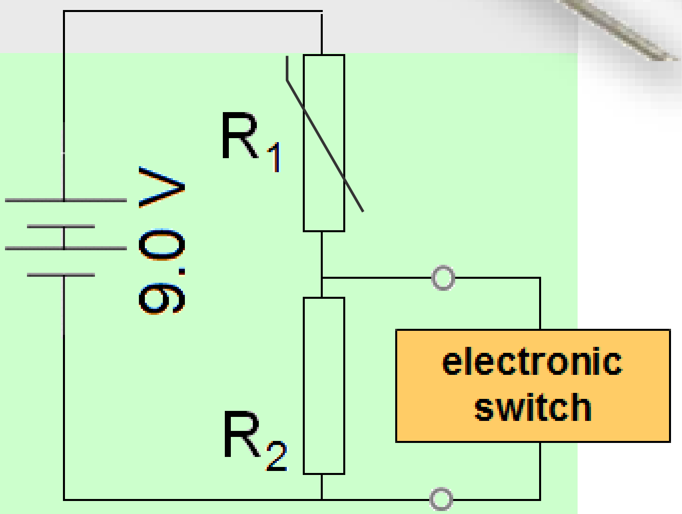
5.2 – Heating effect of electric currents



Potential divider circuits

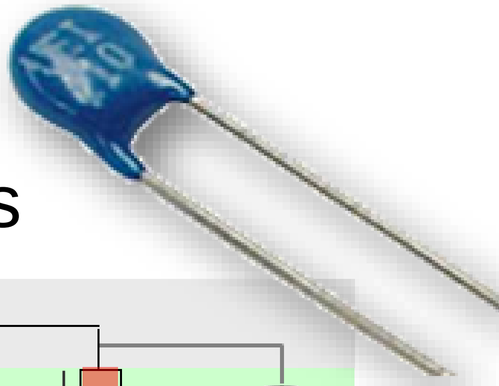
PRACTICE: A thermistor has a resistance of 250 Ω when it is in the heat of a fire and a resistance of 65000 Ω when at room temperature. An electronic switch will turn on a sprinkler system when its p.d. is above 7.0 V.



(a) Should the thermistor be R_1 or R_2?

SOLUTION:

- Because we want a high voltage at a high temperature, and because the thermistor's resistance decreases with temperature, it should be placed at the R_1 position.



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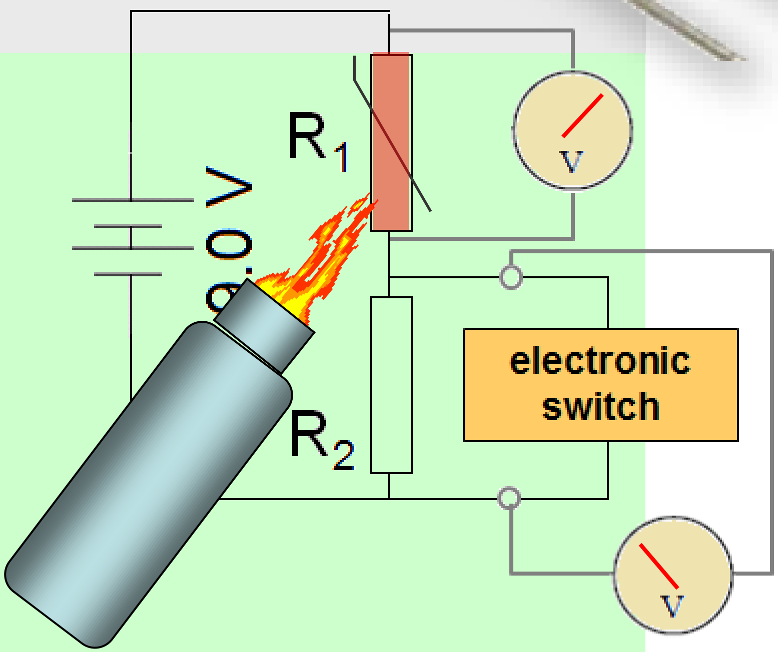
Potential divider circuits



PRACTICE: A thermistor has a resistance of 250 Ω when it is in the heat of a fire and a resistance of 65000 Ω when at room temperature. An electronic switch will turn on a sprinkler system when its p.d. is above 7.0 V.

(b) What should R_2 be?

SOLUTION: In fire the thermistor is $R_1 = 250\ \Omega$.

$$7 = 9\ [\ R_2 / (250 + R_2)\ ]$$

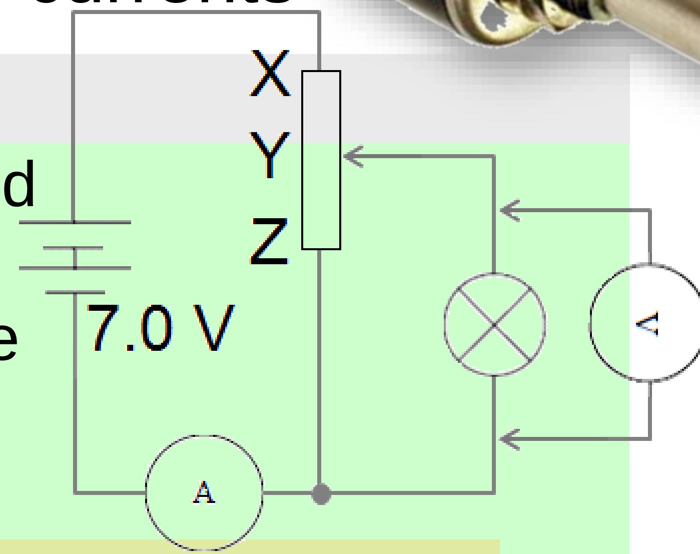
$$7(250 + R_2) = 9R_2$$

$$1750 + 7R_2 = 9R_2 \quad \rightarrow \quad R_2 = 880\ \Omega\ \ (875)$$

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

Potential divider circuits

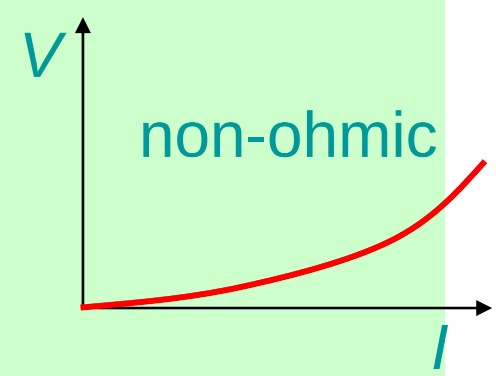
PRACTICE: A filament lamp is rated at “4.0 V, 0.80 W” on its package. The potentiometer has a resistance from X to Z of 24 Ω and has linear variation.



(a) Sketch the variation of the p.d. *V* vs. the current *I* for a typical filament lamp. Is it ohmic? ohmic means linear

SOLUTION: Since the temperature increases with the current, so does the resistance.

- But from *V* = *IR* we see that *R* = *V* / *I*, which is the slope.
- Thus the slope should increase with *I*.



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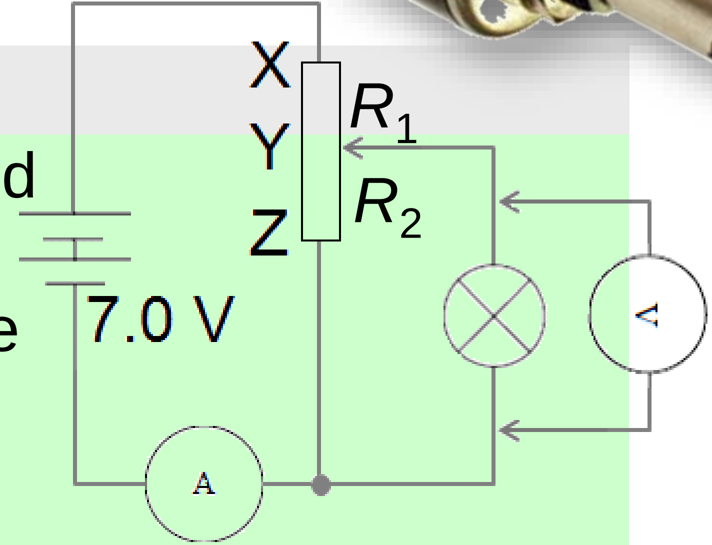
Potential divider circuits



PRACTICE: A filament lamp is rated at “4.0 V, 0.80 W” on its package. The potentiometer has a resistance from X to Z of 24 Ω and has linear variation.

(b) The potentiometer is adjusted so that the meter shows 4.0 V. Will it’s contact be above Y, below Y, or exactly on Y?

SOLUTION: The circuit is acting like a potential divider with R_1 being the resistance between X and Y and R_2 being the resistance between Y and Z.

•Since we need V_{OUT} = 4 V, and since V_{IN} = 7 V, the contact must be adjusted above the Y.

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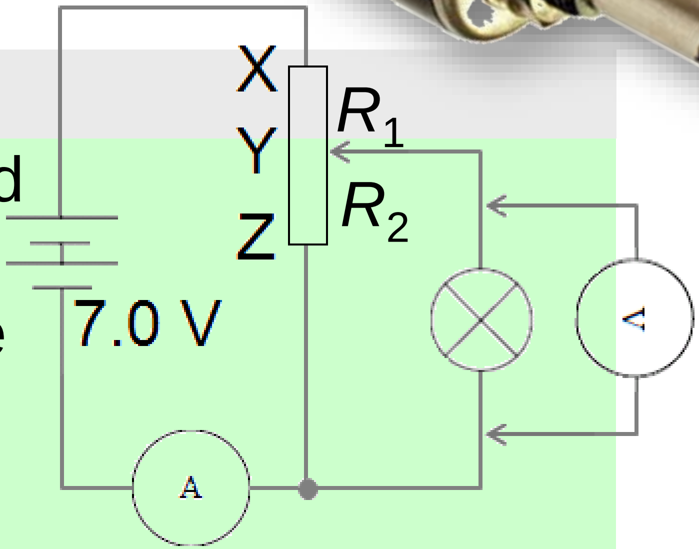
Potential divider circuits



PRACTICE: A filament lamp is rated at "4.0 V, 0.80 W" on its package. The potentiometer has a resistance from X to Z of 24 Ω and has linear variation.

(c) The potentiometer is adjusted so that the meter shows 4.0 V. What are the current and the resistance of the lamp at this instant?

SOLUTION: $P = 0.80$ W and $V = 4.0$ V.

- From $P = IV$ we get $0.8 = I(4)$ so that $I = 0.20$ A.
- From $V = IR$ we get $4 = 0.2R$ so that $R = 20.$ Ω.
- You could also use $P = I^2R$ for this last one.

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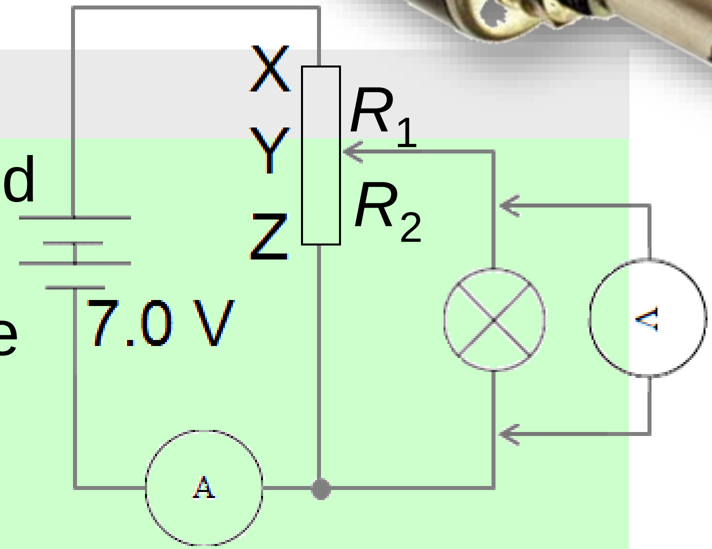
Potential divider circuits



PRACTICE: A filament lamp is rated at “4.0 V, 0.80 W” on its package. The potentiometer has a resistance from X to Z of 24 Ω and has linear variation.

(d) The potentiometer is adjusted so that the meter shows 4.0 V. What is the resistance of the Y-Z portion of the potentiometer?

SOLUTION: Let R_1 = X to Y and R_2 = Y to Z resistance.

- Then $R_1 + R_2 = 24$ so that $R_1 = 24 - R_2$.
- From $V_{OUT} = V_{IN}\,[\,R_2 / (R_1 + R_2)\,]$ we get

$$4 = 7\,[\,R_2 / (24 - \cancel{R_2} + \cancel{R_2})\,] \rightarrow R_2 = 14\ \Omega\ \ (13.71).$$

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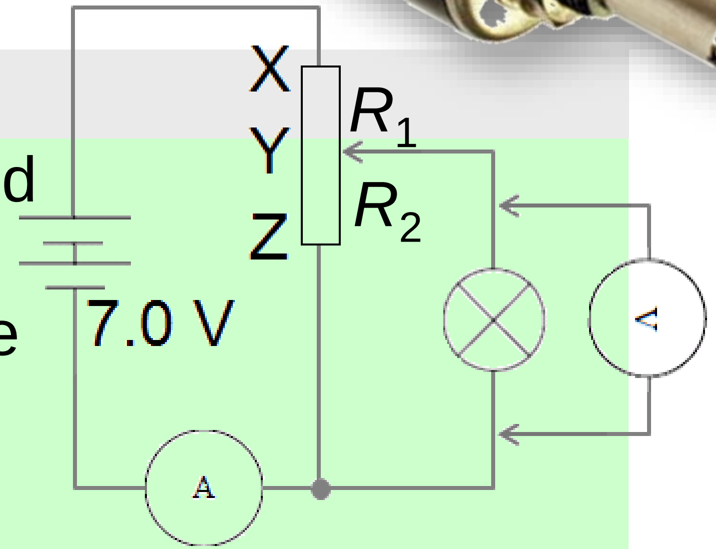
5.2 – Heating effect of electric currents

Potential divider circuits

PRACTICE: A filament lamp is rated at "4.0 V, 0.80 W" on its package. The potentiometer has a resistance from X to Z of 24 Ω and has linear variation.



(e) The potentiometer is adjusted so that the meter shows 4.0 V. What is the current in the Y-Z portion of the potentiometer?

SOLUTION:

•V_2 = 4.0 V because it is in parallel with the lamp.

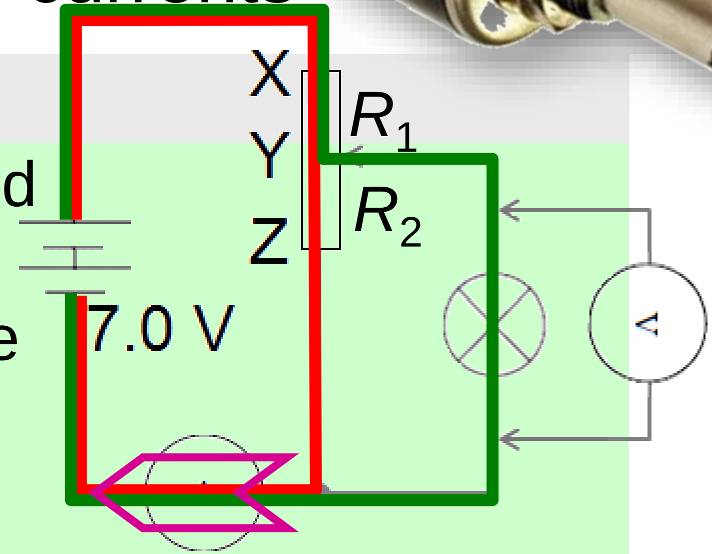
$$I_2 = V_2 / R_2$$
$$= 4 / 13.71 = 0.29 \text{ A}$$

Topic 5: Electricity and magnetism

5.2 – Heating effect of electric currents

Potential divider circuits

PRACTICE: A filament lamp is rated at “4.0 V, 0.80 W” on its package. The potentiometer has a resistance from X to Z of 24 Ω and has linear variation.



(f) The potentiometer is adjusted so that the meter shows 4.0 V. What is the current in the ammeter?

SOLUTION: The battery supplies two currents.

- The red current is 0.29 A because it is the I_2 we just calculated in (e).
- The green current is 0.20 A found in (c).
- The ammeter has both so I = 0.29 + 0.20 = 0.49 A.

Topic 5: Electricity and magnetism
5.2 – Heating effect of electric currents

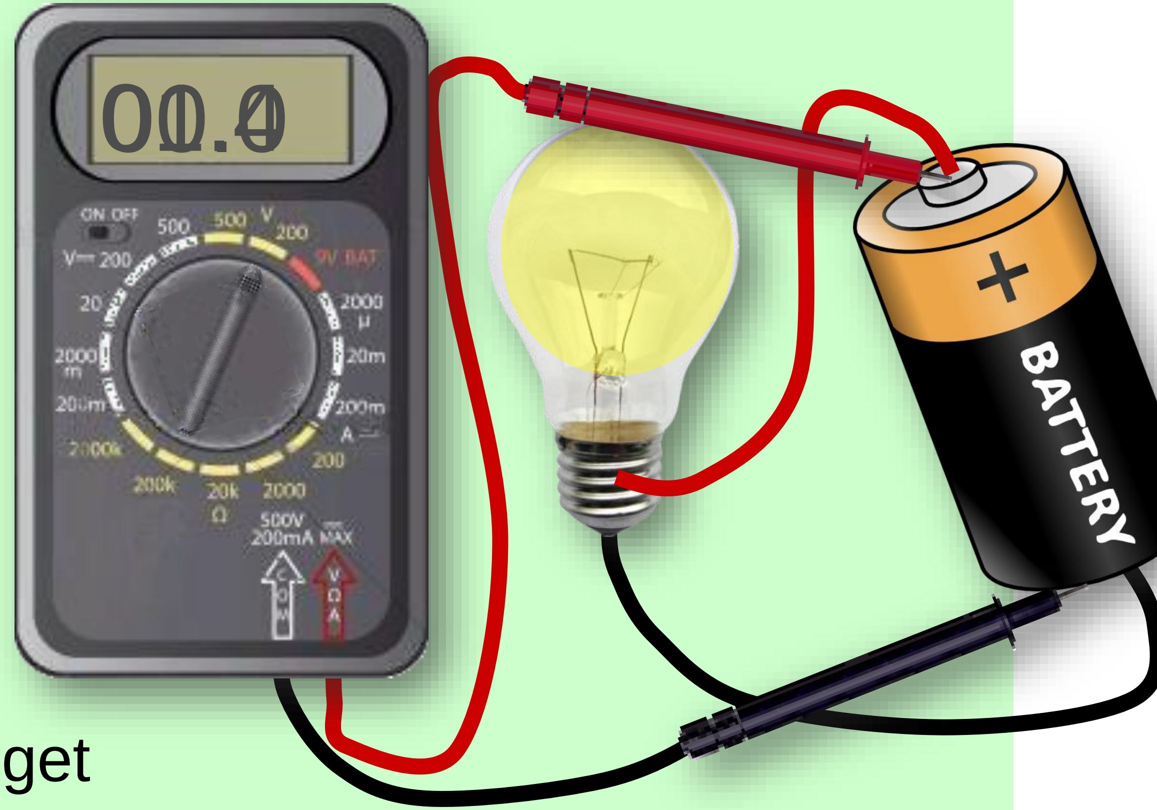
Solving problems involving circuits

PRACTICE: A battery is connected to a 25-W lamp as shown.

What is the lamp's resistance?

SOLUTION:

Suppose we connect a voltmeter to the circuit.



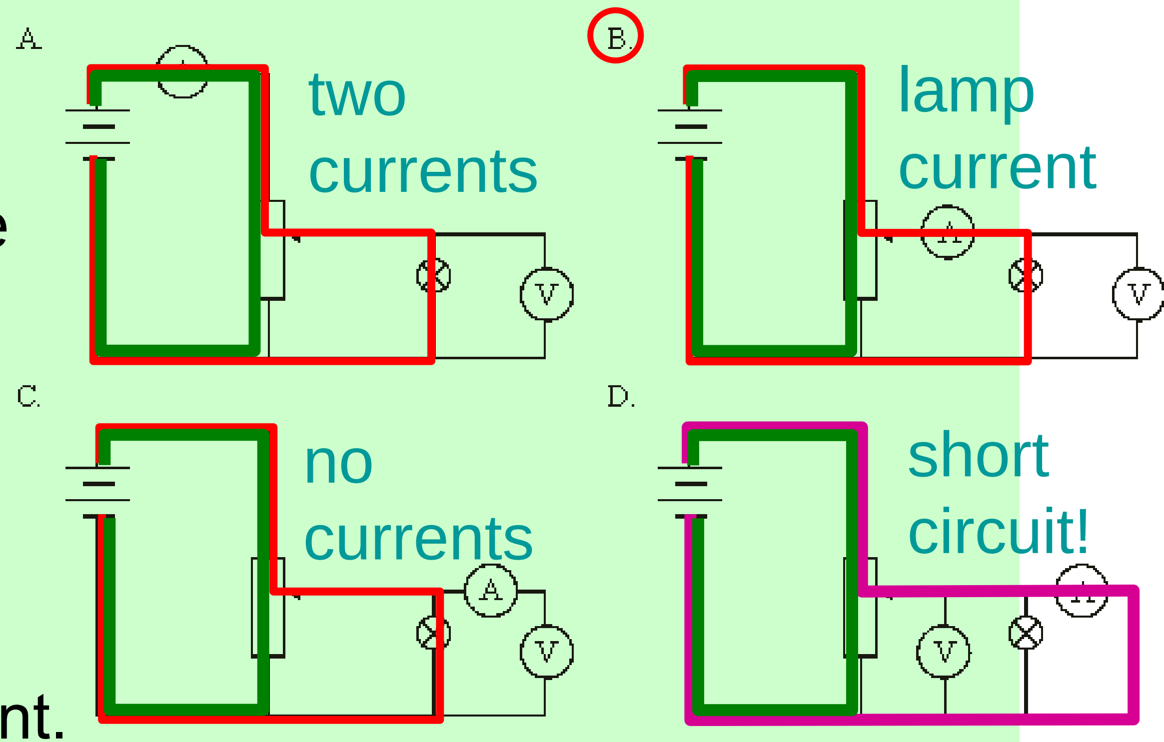
- We know P = 25 W.
- We know V = 1.4 V.
- From $P = V^2 / R$ we get
- $R = V^2 / P = 1.4^2 / 25 = 0.078\ \Omega$.

Topic 5: Electricity and magnetism

5.2 – Heating effect of electric currents

Solving problems involving circuits

PRACTICE: Which circuit shows the correct setup to find the V-I characteristics of a filament lamp?

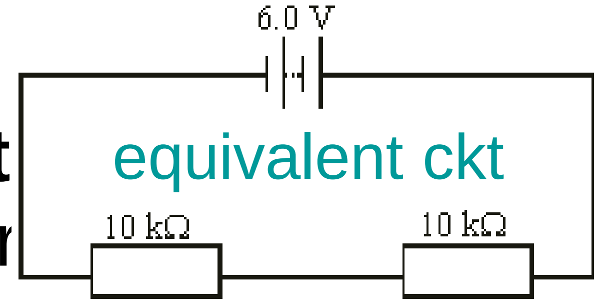


SOLUTION:

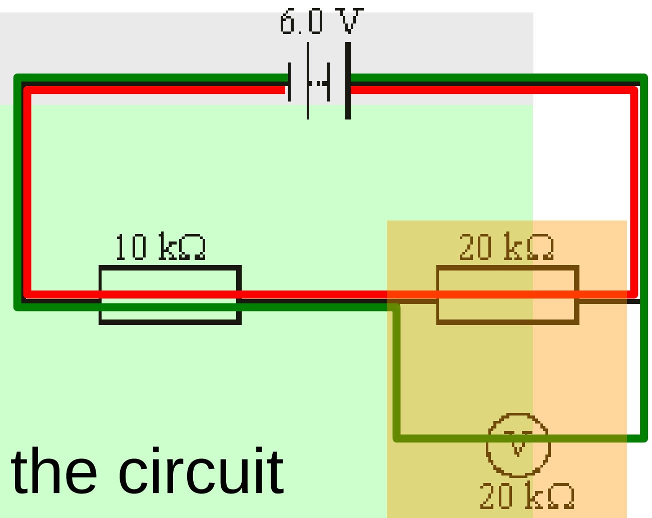
- The voltmeter must be in parallel with the lamp.
- It IS, in ALL cases.
- The ammeter must be in series with the lamp and must read only the lamp's current.
- The correct response is B.

Topic 5: Electricity and magnet

5.2 – Heating effect of electric cur



Solving problems involving circuits



PRACTICE: A non-ideal voltmeter is used to measure the p.d. of the 20 kΩ resistor as shown. What will its reading be?

SOLUTION: There are two currents in the circuit because the voltmeter does not have a high enough resistance to prevent the green one from flowing.

- The 20 kΩ resistor is in parallel with the 20 kΩ so that

$$1 / R = 1 / 20000 + 1 / 20000 = 2 / 20000.$$

$$R = 20000 / 2 = 10\text{ k}\Omega.$$

- But then we have two 10 kΩ resistors in series and each takes half the battery voltage, or 3 V.

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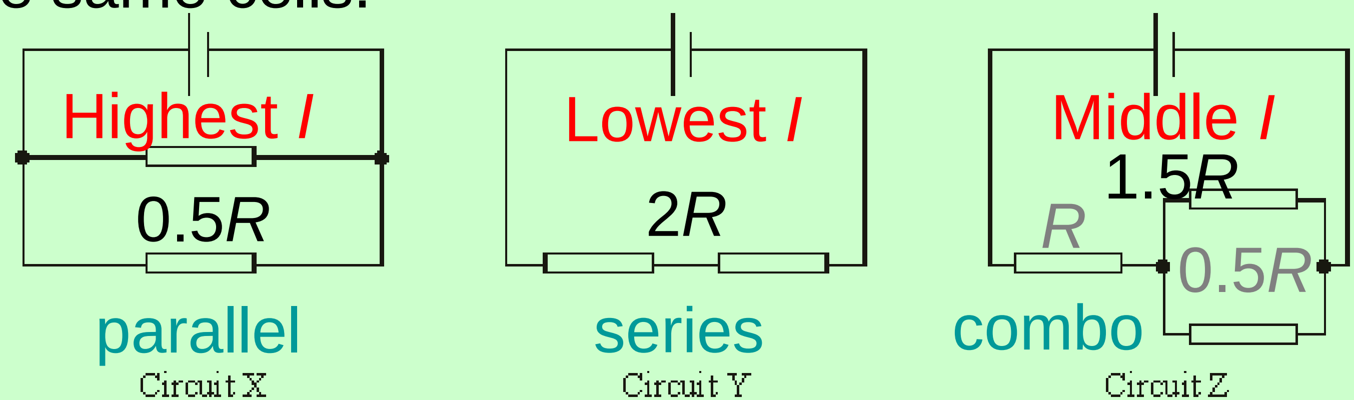
Solving problems involving circuits

PRACTICE: All three circuits use the same resistors and the same cells.



Which one of the following shows the correct ranking for the currents passing through the cells?

	Lowest current	→	Highest current
A.	X	Y	Z
B.	Z	X	Y
C.	Y	Z	X
D.	Y	X	Z

SOLUTION: The bigger the *R* the smaller the *I*.

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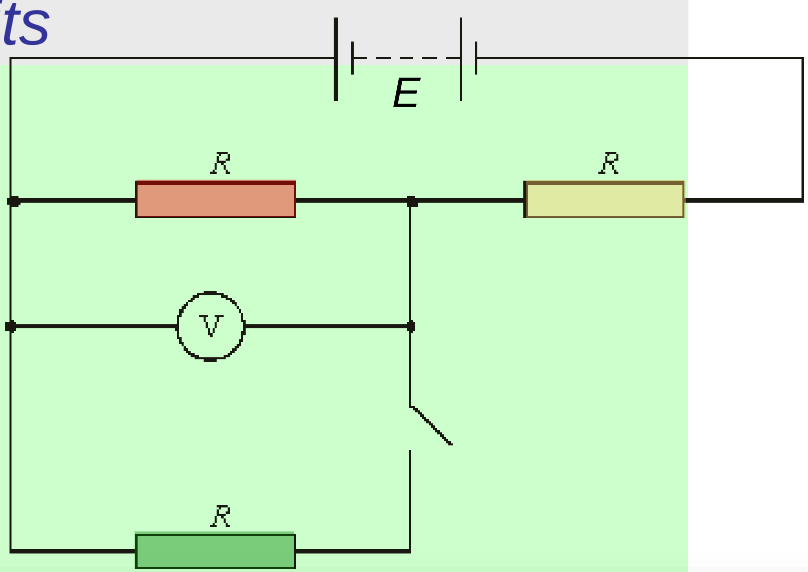
Solving problems involving circuits

PRACTICE: The voltmeter has infinite resistance.

What are the readings on the voltmeter when the switch is open and closed?



	Switch open	Switch closed
A.	0	less than ½ E
B.	0	½ E
C.	½ E	less than ½ E
D.	½ E	½ E

SOLUTION:

- With the <u>switch open</u> the green R is not part of the circuit. Red and orange split the battery emf.
- With the <u>switch closed</u> the red and green are in parallel and are $(1/2)R$.

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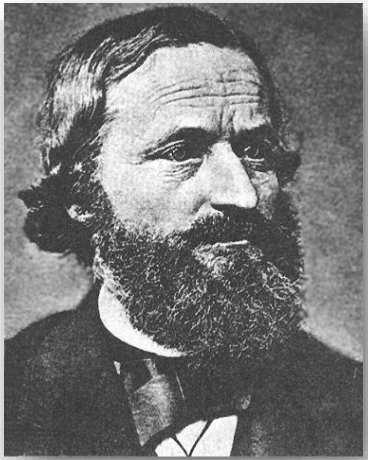
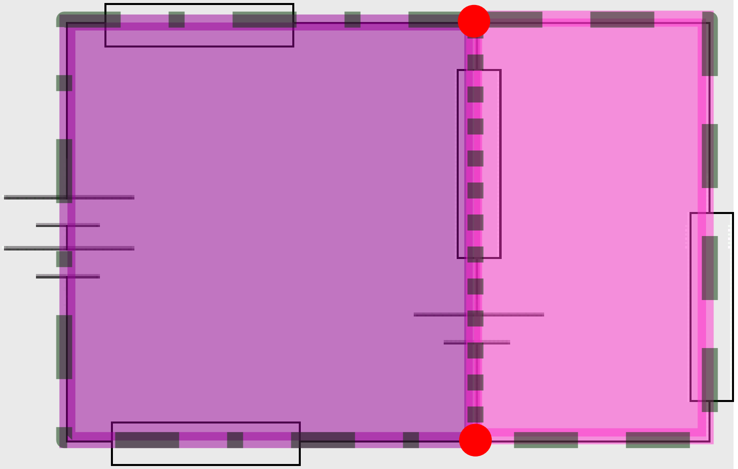
Kirchhoff's rules – junction, branch, and loop

- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- Consider the following circuit containing a few batteries and resistors:
- A **junction** is a point in a circuit where three or more wires are connected together.
- A **branch** is all the wire and all the components connecting one junction to another.
- A **loop** is all the wire and all the components in a complete circle.



Gustav Robert Kirchhoff



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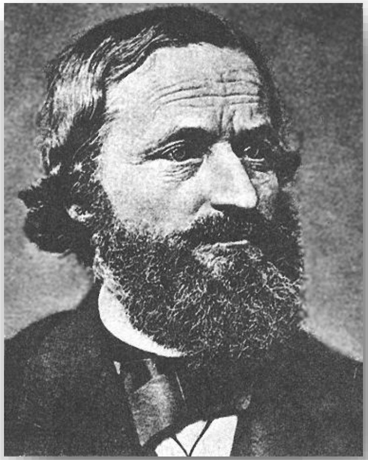
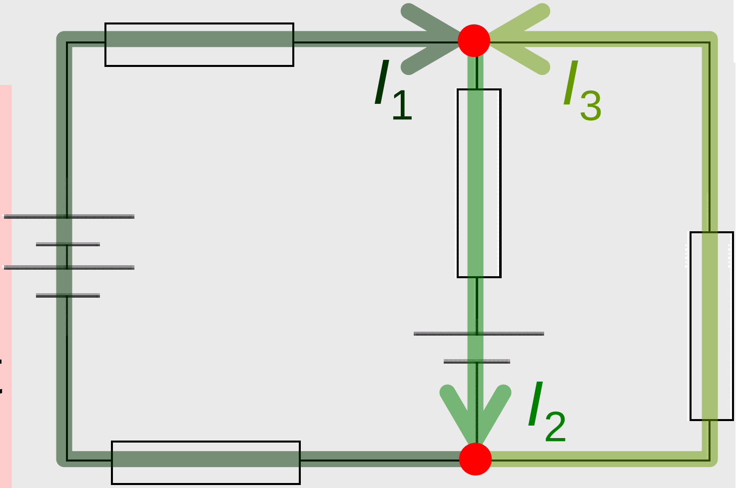


Kirchhoff's rules – the rule for current *I*

- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- STEP 1: Assign a current to each branch.
- If you have a good idea which way it flows, choose that direction.
- If you don't, an arbitrary direction will do just fine.

FYI

- If a current turns out to have a negative solution, you will interpret that as meaning that it flows in the opposite direction.



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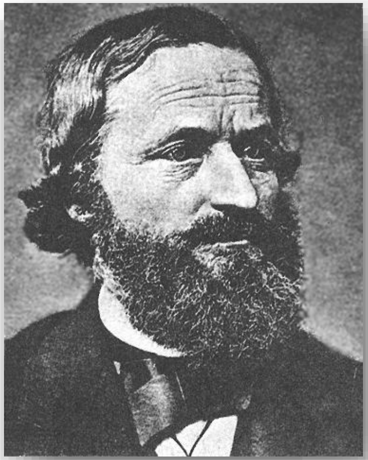
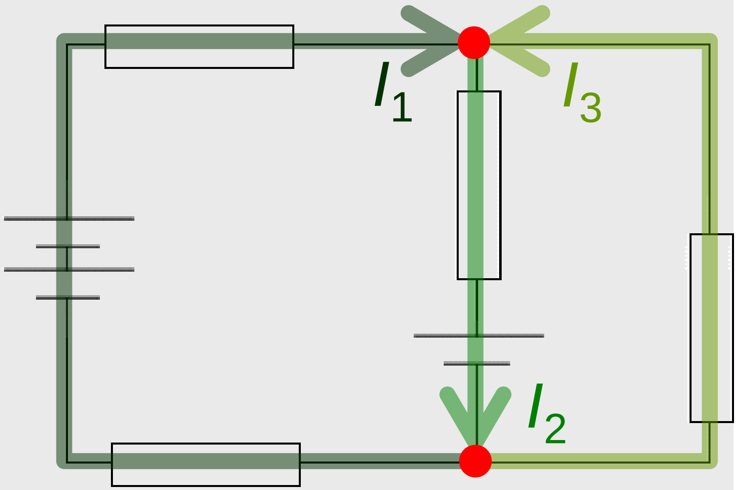


Kirchhoff's rules – the rule for current I

- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- If a current enters a junction it is a "gain" and is assigned a POSITIVE value. If a current leaves a junction it is a "loss" and is assigned a NEGATIVE value.
- For the TOP junction, I_1 and I_3 are both POSITIVE and I_2 is NEGATIVE.
- For the BOTTOM junction, I_1 and I_3 are both NEGATIVE and I_2 is POSITIVE.



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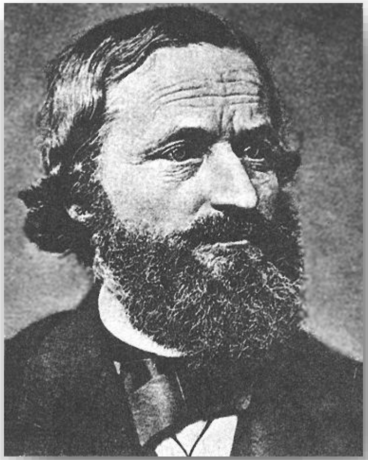


Kirchhoff's rules – the rule for current I

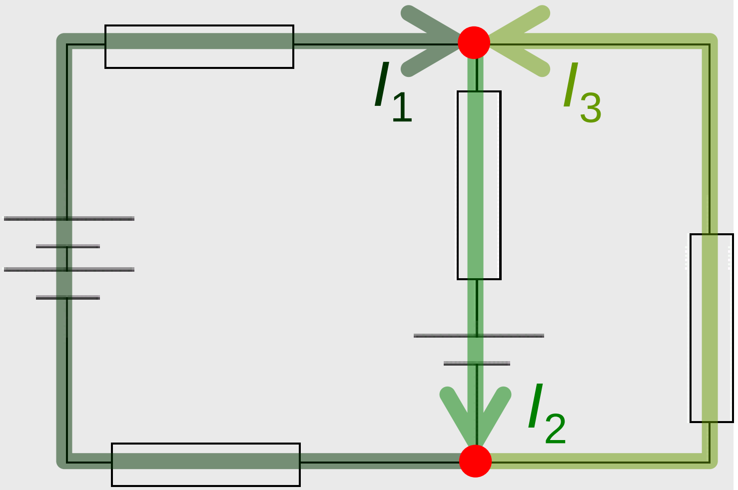
- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- From **conservation of charge** the sum of the currents at each junction is zero.

$\Sigma I = 0$ (junction)	Kirchhoff's rule for I

- STEP 2: Use Kirchhoff's rule for I for each junction.
- Each JUNCTION yields its own equation:

TOP: $I_1 - I_2 + I_3 = 0.$

BOTTOM: $I_2 - I_1 - I_3 = 0.$



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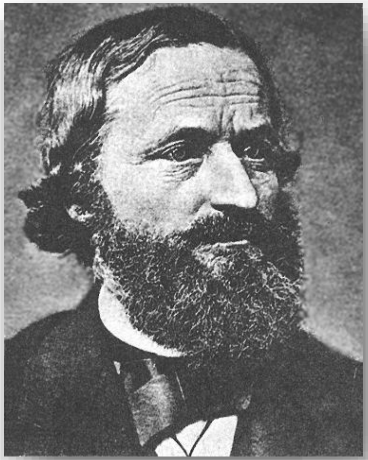
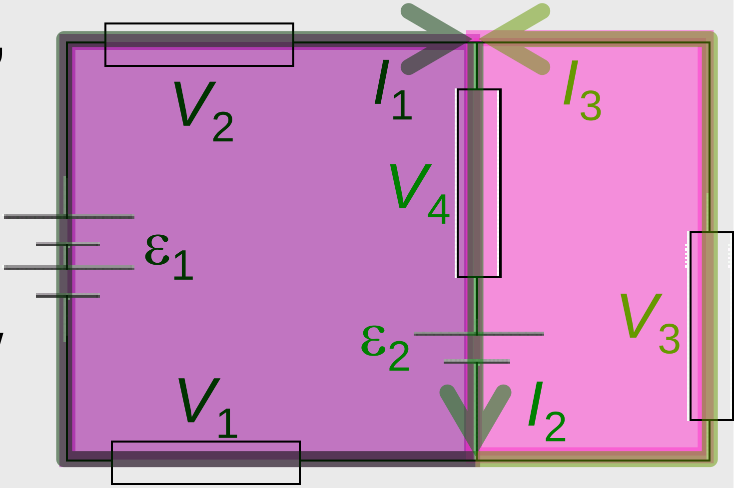


Kirchhoff's rules – the rule for voltage V

- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- Give each resistor a voltage V and each cell or battery an emf ε.
- Cells and batteries increase the energy of the current, resistors decrease the energy.
- Since the energy is qV (or $q\varepsilon$), if we sum up all the energy gains and losses in each LOOP we must get zero.
- We can go either CW or CCW – it doesn't matter.



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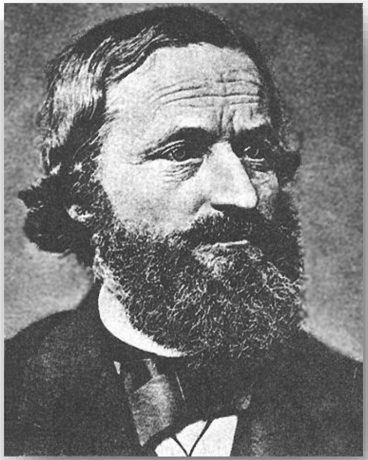
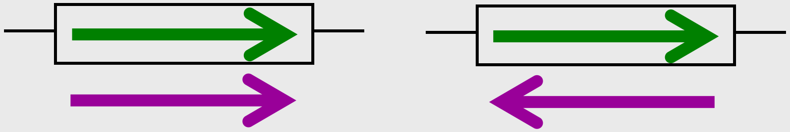


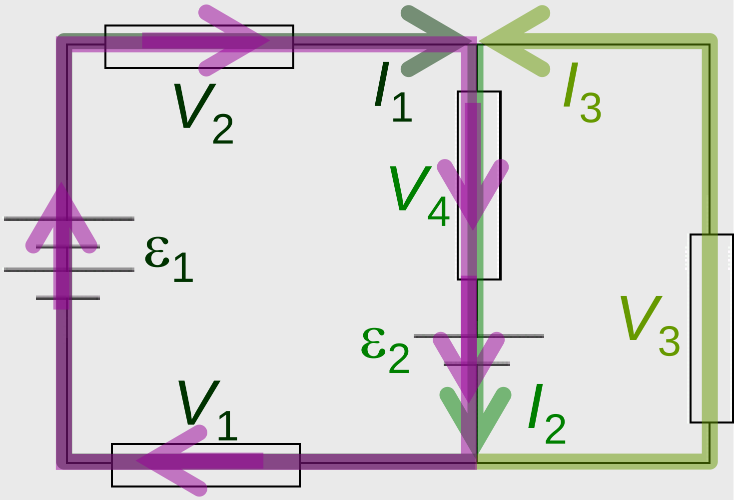
Kirchhoff's rules – the rule for voltage V

•"Solving" a circuit consists of finding the voltages and currents of all of its components.

•If our loop goes in the direction of the branch current assigned earlier, the resistor energy change is negative.

•If our loop goes against the branch current, the sign is reversed for the resistor.





•For our loop we see that we have $-qV_1$ and $-qV_2$ and $-qV_4$.

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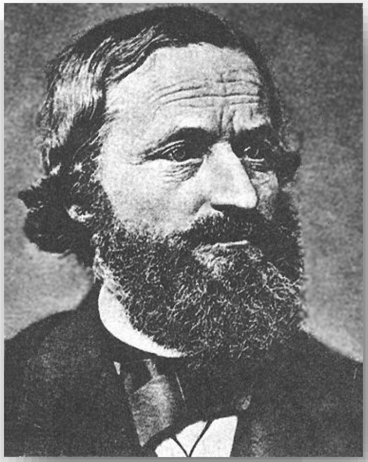
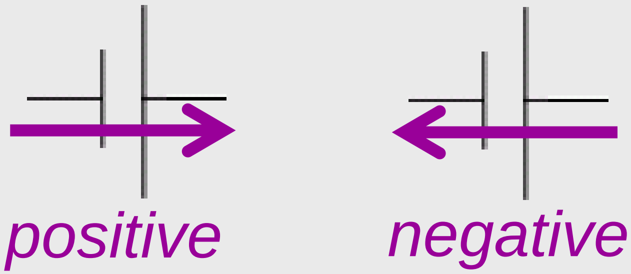
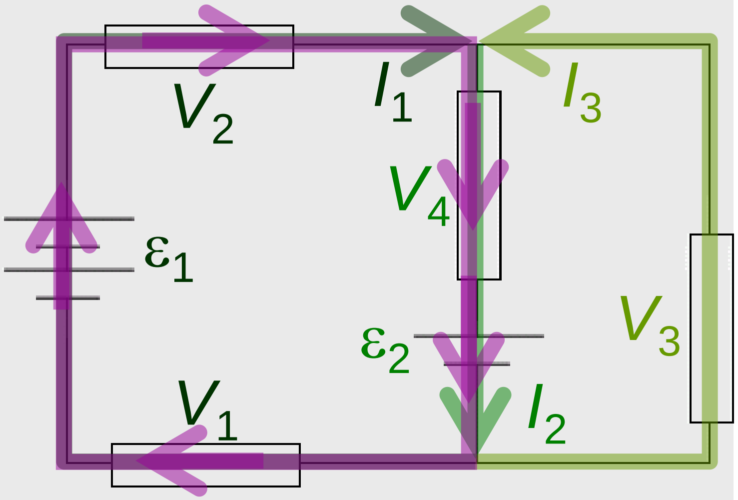
Kirchhoff's rules – the rule for voltage *V*



- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- If our loop goes through a cell from negative to positive, the cell energy change is positive.
- If our loop goes from positive to negative through a cell the energy change is negative.





- For our loop we see that we have $+q\varepsilon_1$ and $-q\varepsilon_2$.

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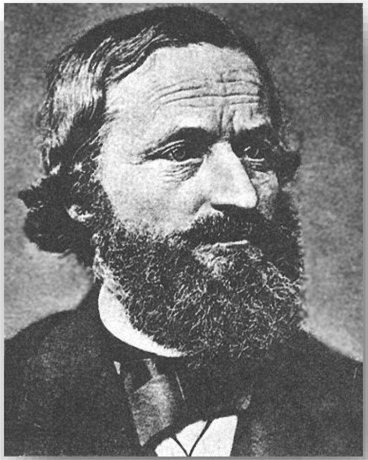
Kirchhoff's rules – the rule for voltage V

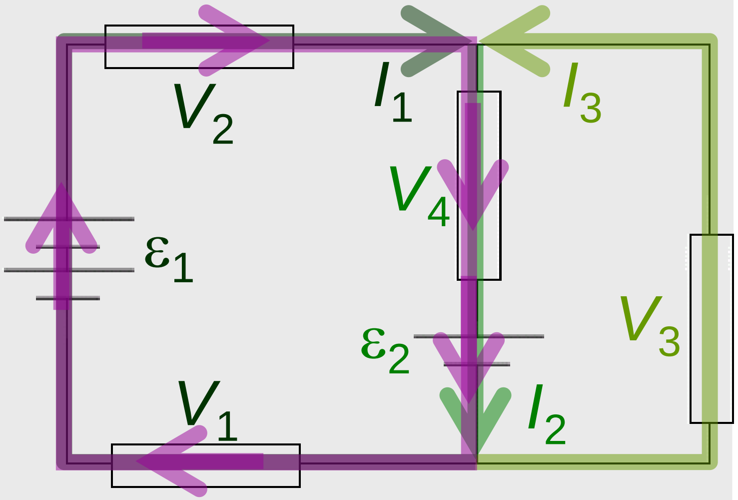


- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- From **conservation of energy** the sum of the voltages in each loop is zero.

$\Sigma V = 0$ (loop)	Kirchhoff's rule for V

- STEP 3: Use Kirchhoff's rule for V for each loop.
- For our loop we have

$$^{-}V_1 + \varepsilon_1 + {}^{-}V_2 + {}^{-}V_4 + {}^{-}\varepsilon_2 = 0.$$



FYI

- The qs all cancel.

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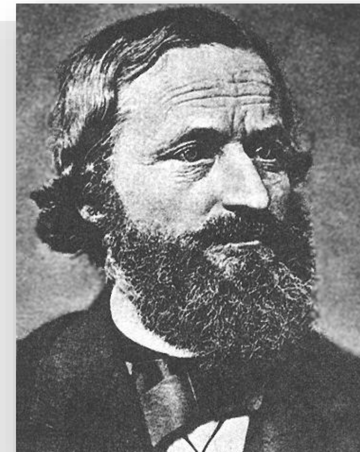
Kirchhoff's rules – the rule for voltage V

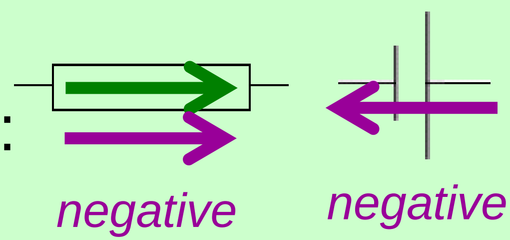


- "Solving" a circuit consists of finding the voltages and currents of all of its components.
- From **conservation of energy** the sum of the voltages in each loop is zero.

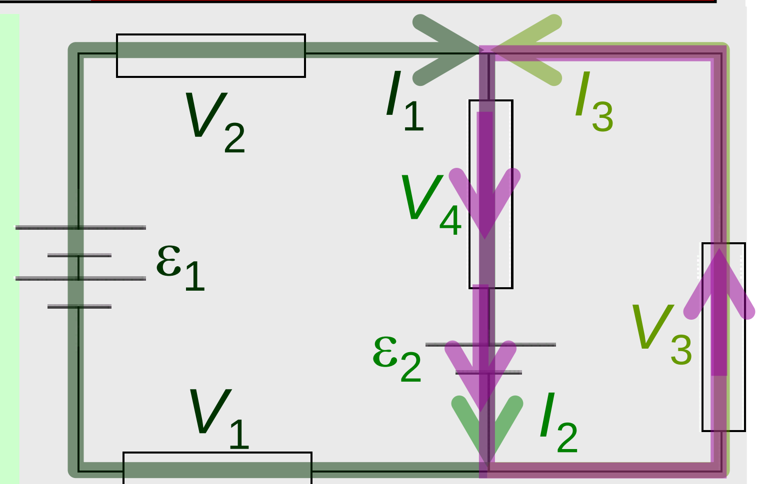
$\Sigma V = 0$ (loop)	Kirchhoff's rule for V

PRACTICE: Using the voltage rule write the equation for the other loop.

SOLUTION:



$$^{-}\varepsilon_2 + {}^{-}V_3 + {}^{-}V_4 = 0.$$



Topic 5: Electricity and magnetism

5.2 – Heating effect of electric currents

Kirchhoff's rules – solving the circuit

EXAMPLE: Suppose each of the resistors is $R = 2.0\ \Omega$, and the emfs are $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6.0$ V. Find the voltages and the currents of the circuit.

SOLUTION: Use Ohm's law: $V = IR$ for the resistors.

- From the rule for I we have $I_1 - I_2 + I_3 = 0$.
- From the rule for V we have

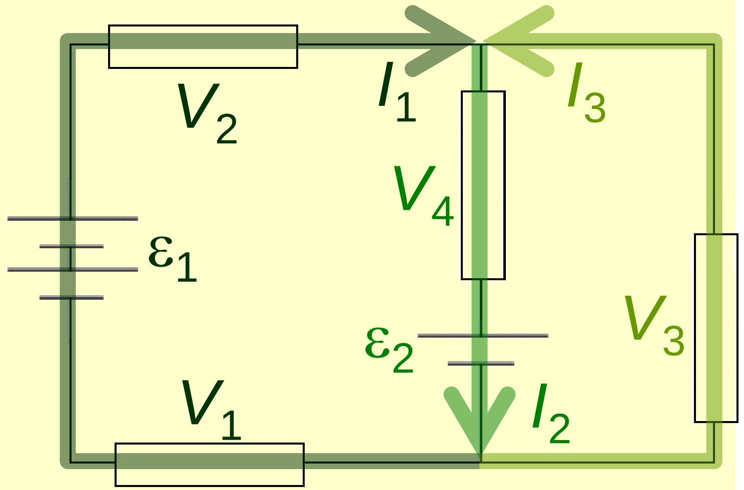
$$^{-}V_1 + {}^{-}V_2 + {}^{-}V_4 + \varepsilon_1 + {}^{-}\varepsilon_2 = 0,$$

$$^{-}\varepsilon_2 - V_3 - V_4 = 0.$$

- From Ohm's law we have

$$^{-}2I_1 + {}^{-}2I_1 + {}^{-}2I_2 + 12 + {}^{-}6 = 0$$

$$^{-}6 - 2I_3 - 2I_2 = 0.$$



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Kirchhoff's rules – solving the circuit

EXAMPLE: Suppose each of the resistors is $R = 2.0\ \Omega$, and the emfs are $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6.0$ V. Find the voltages and the currents of the circuit.

SOLUTION:

- We now have three equations in I.

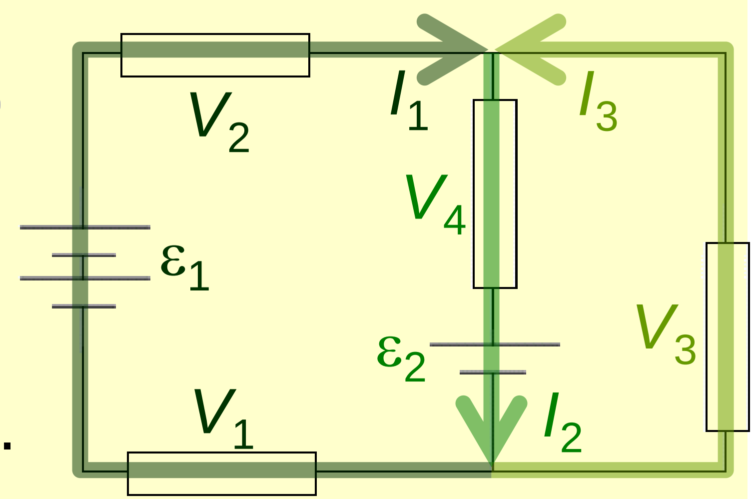
$$I_1 - I_2 + I_3 = 0 \rightarrow I_3 = I_2 - I_1.$$

$${}^{-}2I_1 + {}^{-}2I_1 + {}^{-}2I_2 + 12 + {}^{-}6 = 0$$

$$\rightarrow 6 = 4I_1 + 2I_2 \rightarrow 3 = 2I_1 + I_2.$$

$${}^{-}6 - 2I_3 - 2I_2 = 0$$

$$\rightarrow 6 = -2I_2 + -2I_3 \rightarrow 3 = -I_2 + -I_3.$$



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Kirchhoff's rules – solving the circuit

EXAMPLE: Suppose each of the resistors is $R = 2.0\ \Omega$, and the emfs are $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6.0$ V. Find the voltages and the currents of the circuit.

SOLUTION:

- Now eliminate variables one by one.

(1) $I_3 = I_2 - I_1$. (2) $3 = 2I_1 + I_2$. (3) $3 = -I_2 + -I_3$.

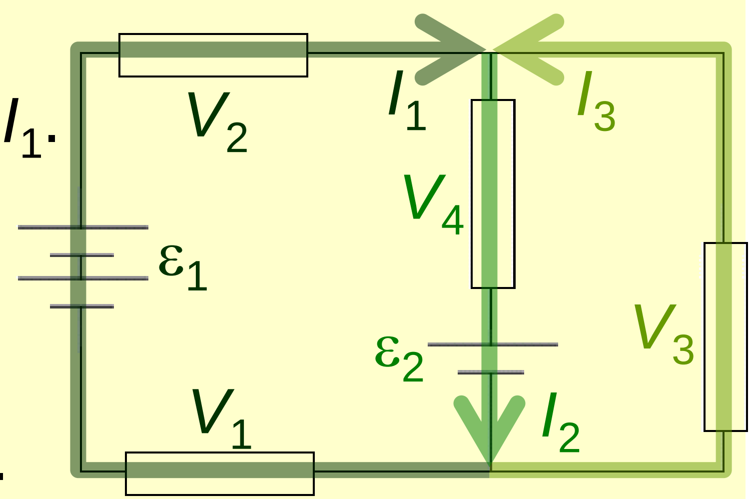
- (1) into (3) eliminates I_3:

$3 = -I_2 + -(I_2 - I_1)$ or $3 = -2I_2 + I_1$.

- Then $I_1 = 3 + 2I_2$.
- Placing into (2) yields

$3 = 2(3 + 2I_2) + I_2$ so that

$3 = 6 + 4I_2 + I_2 \rightarrow I_2 = -0.6$ A.



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Kirchhoff's rules – solving the circuit

EXAMPLE: Suppose each of the resistors is $R = 2.0\ \Omega$, and the emfs are $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6.0$ V. Find the voltages and the currents of the circuit.

SOLUTION:

- Once you have one, you have them all by substitution:

  (1) $I_3 = I_2 - I_1$. (2) $3 = 2I_1 + I_2$. (3) $3 = -I_2 + -I_3$.

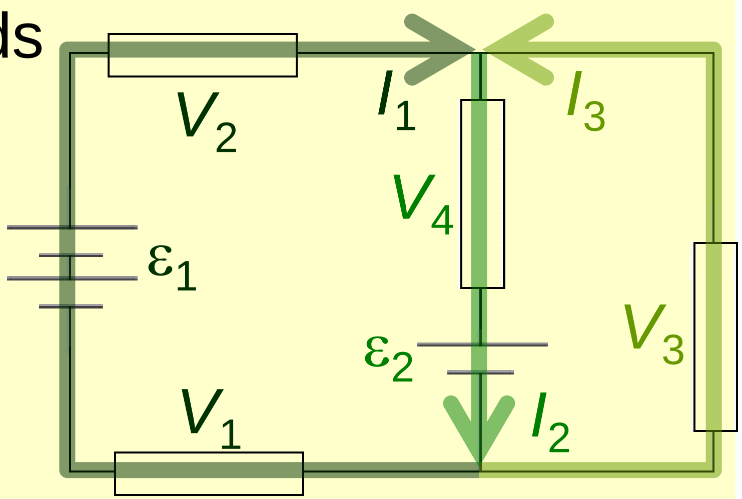
- Putting $I_2 = -0.6$ A into (2) yields

  $3 = 2I_1 + -0.6 \rightarrow I_1 = 1.8$ A.

- Putting I_1 and I_2 into (1) yields

  $I_3 = -0.6 - 1.8 = -2.4$ A.

- Since I_2 and I_3 are negative, we chose the wrong directions.



Topic 5: Electricity and magnetism

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Kirchhoff's rules – solving the circuit

EXAMPLE: Suppose each of the resistors is $R = 2.0\ \Omega$, and the emfs are $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6.0$ V. Find the voltages and the currents of the circuit.

SOLUTION:

- Finally, we can redraw our currents:
- From Ohm's law we calculate our resistor voltages:

$$V_1 = 1.8(2) = 3.6 \text{ V.}$$
$$V_2 = 1.8(2) = 3.6 \text{ V.}$$
$$V_3 = 2.4(2) = 4.8 \text{ V.}$$
$$V_4 = 0.6(2) = 1.2 \text{ V.}$$

- Use both of Kirchhoff's rules to check junctions and loops.